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The Flow of Water through the Straits of Dover, Related to Wind and Differences in Sea Level

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THE FLOW OF WATER THROUGH THE STRAITS OF DOVER, RELATED TO WIND AND DIFFERENCES IN SEA LEVEL

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CONTENTS

The difference in electric potential between the water on opposite sides of the English Channel, as recorded on a telephone cable running from St Margaret's Bay to Sangatte, has been used to measure the mean flow of water through the Straits of Dover. The records were calibrated by means of the tidal currents, which were known from previous measurements. A p.d. of 1 V corresponds to a current of about 140 cm/s (2·75 knots), the exact calibration depending on the electrical conductivity of the sea water and having a seasonal variation. Continuous records were obtained during the 15-month period from February 1953 to June 1954. For 4 months, from November 1953 to March 1954, similar records were also obtained on another cable, crossing the southern North Sea from Aldeburgh to Domburg. Fluctuations due to the earth currents associated with geomagnetic disturbances occurred from time to time, but did not usually cause any difficulty in interpreting the records. The residual flow, after eliminating the tidal currents, has been correlated with the local wind in the Straits and the difference in sea level between the eastern part of the English Channel and the southern part of the North Sea, as determined from tide-gauge records. The tidal currents and elevations were eliminated, approximately, by taking means of 25 hourly readings centred at noon, for each day of the period covered by the observations. The greatest daily rates of flow recorded were 79 cm/s (1·53 knots) towards the north-east on 1 November 1953 and 77 cm/s (1·49 knots) towards the south-west on 3 January 1954. For three periods of unusually strong flow, namely, 19 to 24 September 1953, 26 October to 8 November 1953, and 1 to 6 January 1954, a more detailed analysis was made, eliminating the tidal effects by a method previously used in the analysis of storm surges. The results show the existence of 'current surges', the

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peaks of which lag by up to 6 h behind the corresponding maxima in the wind stress or surface gradient producing them. An attempt has been made to relate the empirical results to the dynamics of flow through the Straits. On the assumption that the 25 h means can be regarded as referring to steady-state conditions, values of $4{\cdot}5\times10^{-3}$ for the wind-stress coefficient γ^2 and $3{\cdot}8\times10^{-3}$ for the bottom friction coefficient k have been deduced. These rather high values may be due, in part, to the steady-state assumption not being justified. The general features of the current surges are consistent with the dynamical treatment.

1. Introduction

Currents produced by the wind in shallow seas differ in several respects from those generated in the open ocean. The effects of surface gradients, due to restrictions on the flow imposed by coastal boundaries, and of friction between the water and the sea bed become more important compared with that of the geostrophic acceleration. Another significant

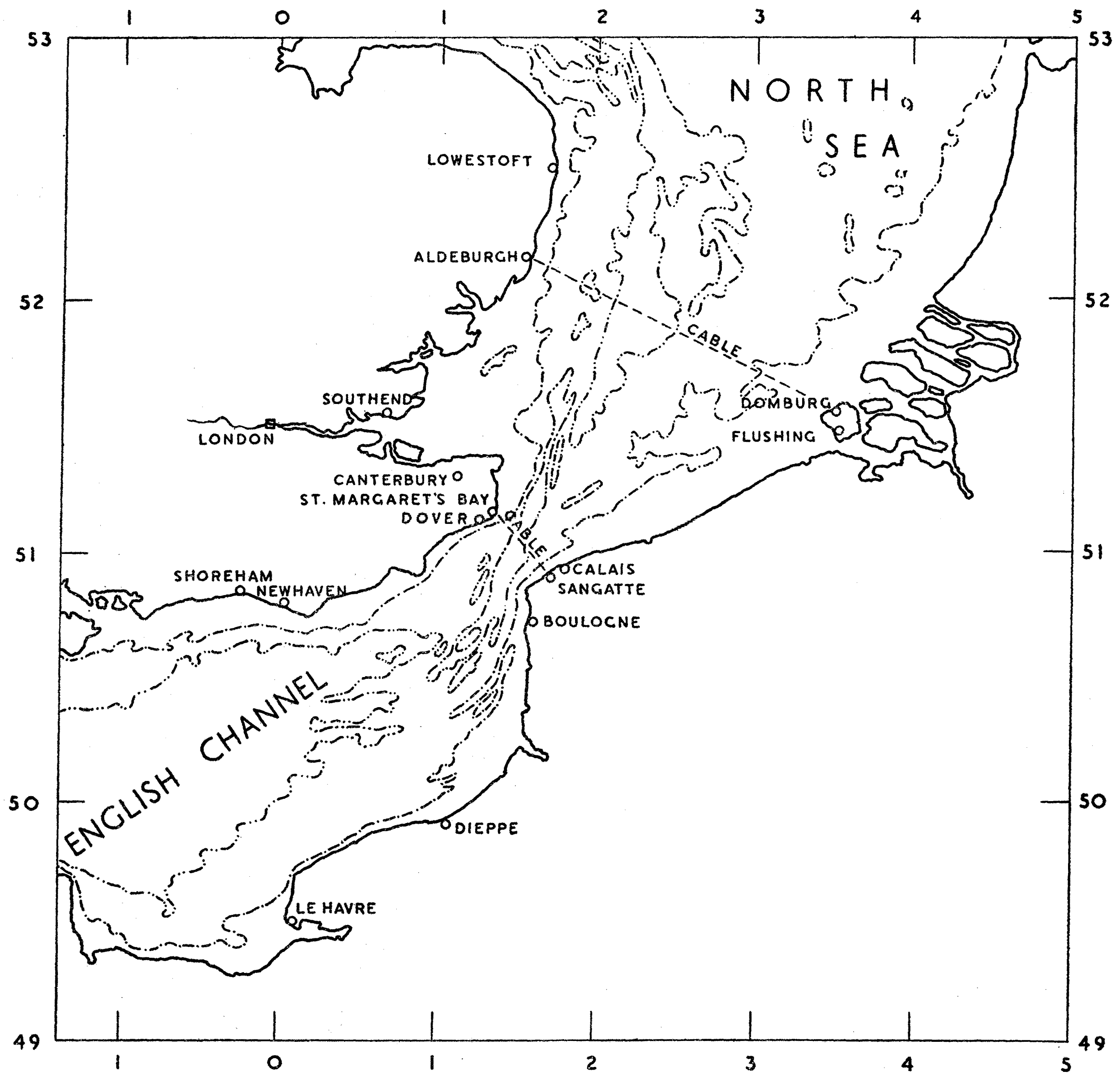

Figure 1. Straits of Dover and adjacent parts of English Channel and North Sea.

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difference is that many coastal seas are normally dominated by strong tidal currents, so that the wind acts on water which is already in vigorous motion.

The flow through the Straits of Dover (see figure 1) provides a good example for an investigation of the physical factors involved in wind currents of this type, and is also a case of practical importance. The exchange of water masses between the English Channel and North Sea, and its variation due to wind and other causes, has been a subject of interest to oceanographers for many years. Among the previous observations may be mentioned the long series of measurements at the *Varne* light-vessel due to Carruthers (1928, 1935 *a*, 1939) and the current measurements at a number of stations across the Straits by Van Veen (1938). Another aspect of the significance of such flow is its association with storm-surge effects in the North Sea and English Channel. The Straits of Dover have been described as acting as a partial safety valve in allowing some escape of water from the North Sea when its level is raised, so causing some reduction in the heights attained in the extreme southern part, including the Thames estuary and the Belgian coast.

The records of flow described in this paper have been obtained from measurements of electric potential differences on a cross-channel telephone cable, which the British Post Office, with the co-operation of the French telephone authorities, very kindly made available for this purpose. This method, as described in § 2, gives the average velocity of flow across a whole cross-section of the Straits, which is a very convenient quantity for this investigation. Since the equipment consists solely of the cable on the sea bed and recording apparatus ashore, it is independent of weather conditions, and records of flow are obtained without difficulty during the stormiest periods, which are, in fact, those of greatest interest. Recordings by this method were continuous for the 15-month period from 24 February 1953 to 4 June 1954, with the exception of two gaps of 12 and 8 days respectively, when the cable had to be taken for telephone operation, owing to faults on other cables. Continuous recordings were resumed on 23 June 1954, after some interruption while work was being done on the cable. The more detailed study of the correlation of the flow with wind and gradients of sea level has been confined to the 7-month period 1 September 1953 to 31 March 1954, but some use has also been made of observations outside this period.

2. The electromagnetic method

Faraday (1832), very soon after his discovery of electromagnetic induction, pointed out that water flowing in a channel would act as a moving conductor, cutting the vertical lines of force of the earth's magnetic field, and would have an electromotive force induced in it. He gave the Straits of Dover as an example, stating that the e.m.f. would change sign when the tidal currents changed direction. Electric currents, alternating with the tidal period, and attributed to the effect of tidal streams, were reported subsequently on a number of submarine cables. These observations have been summarized by Longuet-Higgins (1949). The impetus to the use of the method for the present investigation was given by the observations of Cherry & Stovold (1946), who, in the course of their work of restoring telephone communications between England and the Continent after the War, recorded tidal e.m.f.'s on four submarine cables.

A theoretical treatment of the distribution of electric potential due to e.m.f.'s induced by the flow of water in a channel has been given by Longuet-Higgins (1949). For a shallow channel, i.e. one whose depth is small compared with its width, the following general results apply:

(*a*) The potential difference between points at the same horizontal level depends on the vertical component, Z, only, of the earth's magnetic field.

(*b*) The p.d. due to Z depends only on the mean velocity in each vertical line, regardless of the variation of velocity with depth.

(*c*) The potential distribution depends on the ratio of the electrical conductivity of the water to that of the sea bed and on the relative dimensions of the channel.

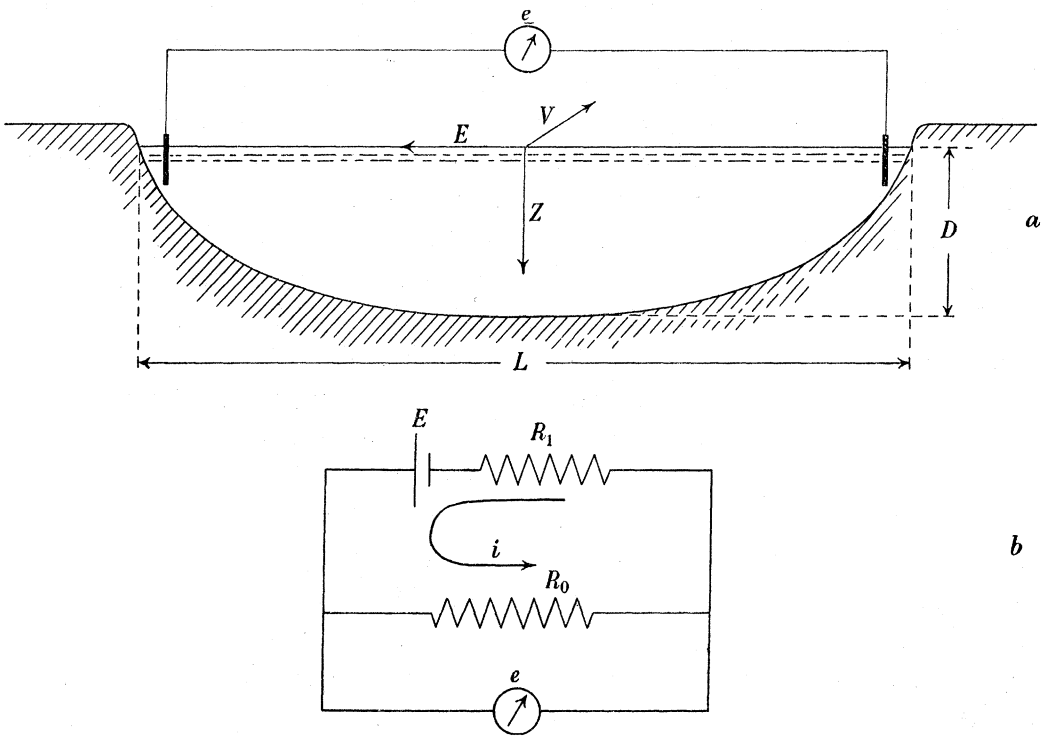

FIGURE 2. *a*, e.m.f. induced in a channel. *b*, equivalent circuit.

Longuet-Higgins treated the third point by considering, as an idealized case, a long straight channel of semi-elliptical cross-section. He showed that the horizontal gradient of potential in the water is given by

$$\frac{\partial \phi}{\partial x} = \frac{VZ \tanh \xi_1}{(\kappa_0/\kappa_1) + \tanh \xi_1}, \tag{1}$$

where x is measured across the channel, V is the velocity of the water along the channel, averaged with respect to depth and assumed to be uniform across the channel, $\tanh \xi_1$ is the ratio of the minor (vertical) to the major (horizontal) axes of the ellipse and κ_1 and κ_0 are the specific conductivities of the water and of the channel bed.

The potential difference e in volts between the two sides of the channel, of width L and maximum depth D, may be written

$$\left.\begin{aligned} & e = \delta E, \\ \text{where} \quad & E = VZL \times 10^{-8} \\ \text{and} \quad & \delta = \left(1 + \frac{\kappa_0}{\kappa_1}\frac{L}{2D}\right)^{-1}, \end{aligned}\right\} \tag{2}$$

Z being assumed constant across the channel, and $\coth \xi_1$ being replaced by $L/2D$. These conditions are illustrated in figure 2*a*. The equivalent circuit is shown in figure 2*b*, where R_1 represents the internal resistance of the sea water and R_0 the resistance of the external circuit through the sea bed. By comparison with equation (2), it is seen that one may put $R_1 = 1/\kappa_1$ and $R_0 = 2D/\kappa_0 L$.

E represents the potential difference which would be measured if the channel bed were non-conducting, and δ the factor by which the measured p.d. is reduced due to the finite conductivity of the bed. It is seen that δ is increased if either (i) the ratio of depth to width of channel is increased, or (ii) the specific conductivity of the water relative to that of the bed is increased. The effect of (i) due to tidal rise and fall may be shown to be small, especially if mean values over one or more tidal periods are taken. The effect of (ii) due to seasonal changes in κ_1 arising from changes in temperature and salinity is appreciable, and has been taken into account in converting measured voltages into velocities. The treatment given by Longuet-Higgins indicates that the return currents in the sea bed extend to a depth comparable with the width of the channel, so that the effective value of κ_0 depends on the conductivity of the rocks to some considerable depth below the channel. This conductivity has not been determined by any other method, and so δ cannot be found from equation (2). The calibration factor, by which the measurements in volts were converted to velocities, was determined quite empirically, as described in § 5·1. This empirical factor could then be used to estimate κ_0, the effective specific conductivity of the sea bed.

3. Apparatus

The measurements were made on a reserve coaxial cable running from St Margaret's Bay, near Dover, to Sangatte, near Calais (figure 1), a distance of 34 km. The outer conductor of the sea cable, of helically wound copper tape, was effectively in contact with the sea water throughout its length, with the exception of the portion of it running from the beach to the repeater station at St Margaret's Bay, which was insulated from earth. At the Sangatte end, the inner and outer conductors were connected together. The recording apparatus, installed in the repeater station at St Margaret's Bay, was connected between the inner and outer conductors at that end. No special electrodes were used, the outer conductor of the cable itself serving the purpose. In the initial stages, some records were obtained with the inner conductor connected to the station earth at the Sangatte end and the recorder connected between the inner conductor and the station earth at St Margaret's Bay. The tidal variations appeared with the same amplitude as when using the sea earth, but there was a difference in the zero potential.

The recording circuit, using a mirror galvanometer, is shown in figure 3*a*. The resistance-capacity circuit offers a high resistance to the cable, so that no possibility of polarization effects due to current taken by the recording circuit can arise, and has a time constant of about 30 s, thus smoothing out the more rapid fluctuations of potential (see § 4). Provision was made (not shown in the figure) for applying a calibrating voltage from a 1·5 V dry cell. Recording was carried out photographically on paper $5\frac{1}{2}$ in. wide, the paper-drive being by synchronous motor and time marks being put on the record every 20 min. Initially a paper speed of 10 in. per hour was used, but this was reduced later to 2 in. per hour.

Some typical records are shown in figure 4. The most conspicuous feature is the tidal variation, sometimes with irregular fluctuations of shorter period superimposed. The lower trace on each record is of the p.d. measured on an inland cable between St Margaret's Bay and Canterbury, for the purpose described in § 4.

Nearly all the measurements on the St Margaret's Bay–Sangatte cable, which are utilized in this paper, were obtained by the above method. For about 4 months, from November 1953 to March 1954, however, recordings were made on another cable, crossing the southern part of the North Sea from Aldeburgh to Domburg, by a different method. This

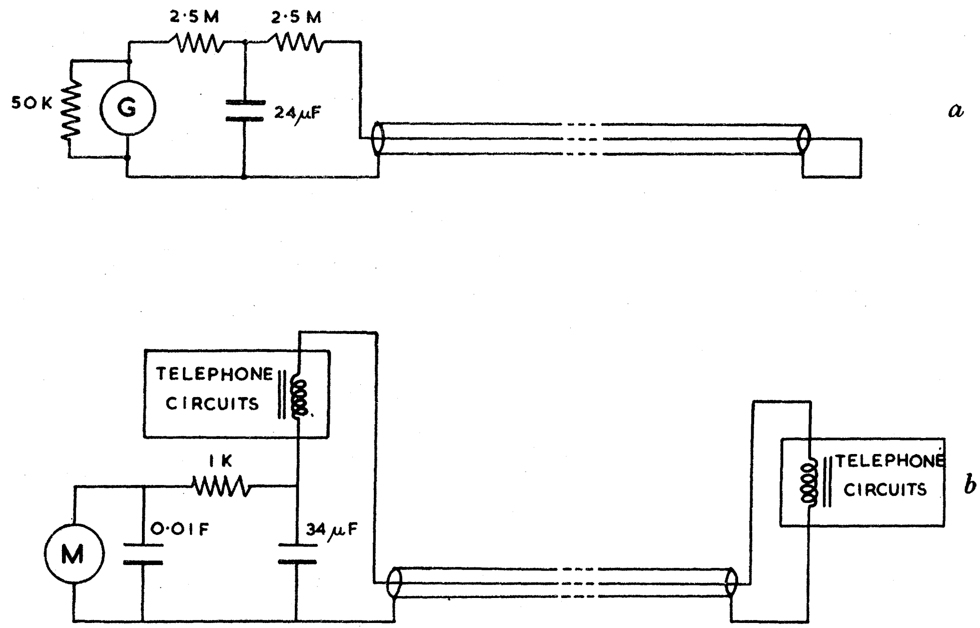

FIGURE 3. Recording circuits. *a*, St Margaret's Bay–Sangatte cable; *b*, Aldeburgh–Domburg cable.

cable, also of the coaxial type, was in continuous use for telephone communications, but it was found possible to record the d.c. potential difference at the same time, with no mutual interference, using the circuit shown in figure 3*b*. The 34μF capacitor provided a low-impedance path to the telephone signals but blocked the d.c. current, which was recorded by the recording milliammeter M, an instrument of the siphon-pen type, having a full-scale deflexion of 1 mA and internal resistance of 1000 Ω. The 0·01 F condenser was inserted to provide a time constant of about 30 s, to reduce the response to short-period fluctuations. Experience at Aldeburgh having shown that the p.d.'s due to water movements could be recorded equally well and more simply using a pen recorder with a resistance of the order of 1000 Ω, a similar recorder was installed at St Margaret's Bay in place of the photographic recorder in April 1954, and has since been providing satisfactory records.

4. Earth currents associated with magnetic disturbances

The records sometimes showed fluctuations of a more or less irregular character and of a wide range of periods, which were attributed to the earth currents associated with magnetic disturbances. The occurrence of such currents is well known, but it was thought necessary to examine them further in this case in order to be able to separate them from variations in p.d. due to variations in the flow of water.

A second recorder was put into operation on a cable running inland from St Margaret's Bay to Canterbury, a distance of 24 km in a direction approximately the same as the cross-Channel cable, i.e. true bearing 132°. A comparison of the two traces showed a general

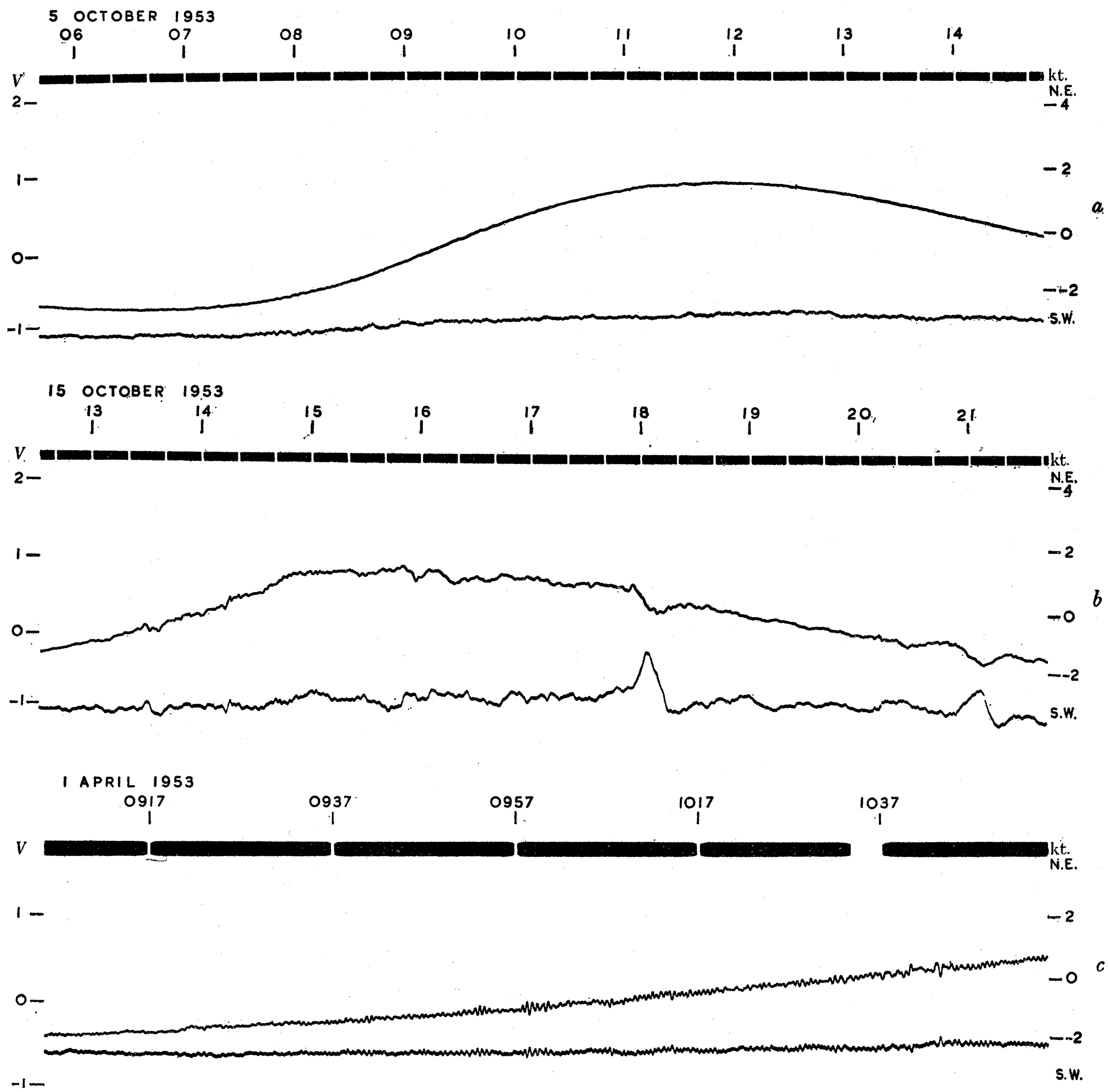

Figure 4. Typical records from St Margaret's Bay–Sangatte cable. *a*, 5 October 1953; *b*, 15 October 1953; *c*, 1 April 1953.

correspondence, which was better in the short periods (of the order of 30 s) than in the longer ones of several minutes or more. An attempt was made to connect the outputs of the two cables in such a way that the fluctuations would be balanced out, while the small tidal p.d. on the inland cable would be added to that on the sea cable. Only partial success was achieved. While the compensation was fairly good for the short-period fluctuations, it had

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little effect on those of longer period, apparently because of differences in phase and waveform. It is possible that the earth currents were not in the same direction relative to the two cables and that the balance might have been improved by adding a proportion of the component of earth current at right angles to the St Margaret's Bay–Canterbury cable, if this had been practicable. The method subsequently adopted was to reduce the response of the recorder to the shorter period fluctuations by inserting a resistance-capacity circuit with a time constant of about 30 s. The fluctuations of longer period were smoothed graphically when reading the records.

As far as is known, no other earth-current measurements were being made in Great Britain at the time, but in Holland earth currents were being recorded on a cable running inland from Amsterdam to Hengelo, a distance of 130 km in a line bearing 110° true. By the courtesy of Dr J. Veldkamp, of the Koninlijk Nederlands Meteorologisch Instituut, some of these records were made available and, for the period from 10.00 h on 21 March to 07.00 h on 25 March 1953, the records from the Dutch cable were examined side by side with the Canterbury–St Margaret's Bay and St Margaret's Bay–Sangatte records. The recorder on the Amsterdam–Hengelo line has a time constant of 20 s, so that the short-period fluctuations were not recorded. The fluctuations of longer period showed close correspondence on all three records, but often with time lags between corresponding peaks. Those on the St Margaret's Bay–Sangatte record occurred later than those on the Canterbury–St Margaret's Bay one by up to 5 min, and those on the Amsterdam–Hengelo record were usually later than on the St Margaret's Bay–Sangatte one by a similar amount, but on one occasion by 10 min. The time lags tended to be greater the longer the period of the fluctuations. Most periods were 20 to 30 min, but that corresponding to the 10 min lag was about 45 min. On the other hand, there was a fluctuation with an abrupt beginning at 01.19 h on 22 March which occurred simultaneously (to within a minute) on all three records. The general direction of the p.d. was always the same on the three lines, i.e. if Canterbury was positive relative to St Margaret's Bay, the latter was positive relative to Sangatte, and Amsterdam was positive relative to Hengelo.

From 27 November 1953 to 31 March 1954, records from both Aldeburgh–Domburg and St Margaret's Bay–Sangatte cables were available and the fluctuations on them were examined side by side. A close correspondence between the variations was apparent, the shape of the traces on the two records being very similar in most cases. The fluctuations occurred within 2 or 3 min of one another, the accuracy of timing on the Aldeburgh–Domburg records not being sufficient to allow a closer comparison. The polarity was always the same on the two cables, i.e. Domburg was positive relative to Aldeburgh when Sangatte was positive relative to St Margaret's Bay. From measurements of the amplitudes of nineteen large peaks occurring between 4 December 1953 and 30 March 1954, the amplitude of disturbance on the Aldeburgh–Domburg cable varied between 2·7 and 5·7 times that of the corresponding disturbance on the St Margaret's Bay–Sangatte cable, the ratio averaging 4·2. This is nearly the same as the ratio of the lengths of the cable, which is 4·5, and indicates that the potential gradient in the sea bed, due to corresponding disturbances, is approximately the same in the two areas. Examination of the records showed a rather closer correspondence between disturbances on the Aldeburgh–Domburg and St Margaret's Bay–Sangatte cables than between those on the St Margaret's Bay–Canterbury and

St Margaret's Bay–Sangatte cables. Figure 5 shows an example of corresponding disturbances on the records from the three cables.

A comparison of some of the early records on the St Margaret's Bay–Sangatte cable with the magnetic variations recorded at Abinger was made with the valuable co-operation of Mr H. E. Finch, of the Royal Observatory, Herstmonceux. For a number of days between 12 February and 28 February 1953, the cable records were examined side by side with the

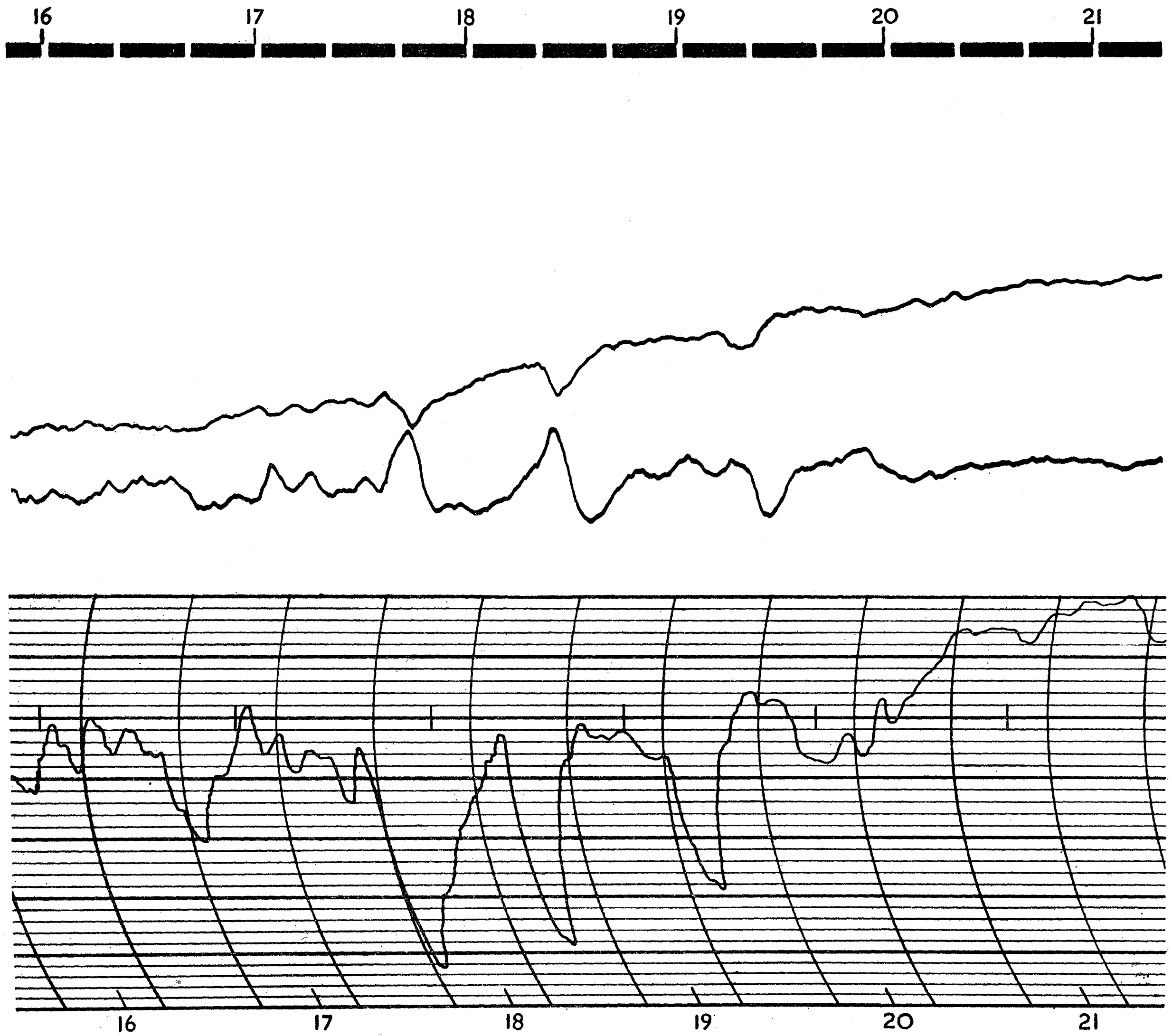

Figure 5. Corresponding disturbances on three cables, 16.00 h to 21.00 h 14 March 1954. Above: St Margaret's Bay–Sangatte and St Margaret's Bay–Canterbury. Below: Aldeburgh–Domburg.

records of variations in the total horizontal component H, the declination D and vertical component Z of the geomagnetic field. Good qualitative agreement was found, every fluctuation on the cable record corresponding to fluctuations in one or more of the magnetic components: most commonly in H, often in D, and less frequently in Z. The correspondence was apparent in short-period bursts of fluctuations (period 20 to 30 s) and also for slower fluctuations with periods up to 30 min. Some of the slower fluctuations on the magnetic records, however, did not correspond to any movement on the cable record. The ratio of

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the amplitudes of the magnetic and potential difference fluctuations was of the same order for long- and short-period disturbances.

The fluctuations were not investigated more closely, as it appeared to be well enough established that they were associated with magnetic disturbances and were not due to variations in the flow of water across the section traversed by the cable.

5. Methods of analysis of the records

The displacement of the trace from the zero line on the record was measured at hourly intervals and the distances in millimetres converted to potential differences in volts, using the calibration signal. The mean of values over a long period was taken as the p.d. corresponding to zero drift current, and this mean value was subtracted from each hourly reading. To convert the p.d.'s to velocities of flow, a calibration based on the tidal currents was used.

5·1. *Tidal currents*

Using 29 days' observations, the readings were tabulated according to the nearest hour before or after high water at Dover, and the mean value for each hour computed. These mean values were multiplied by 1·33 to convert from mean current to spring rates, assuming

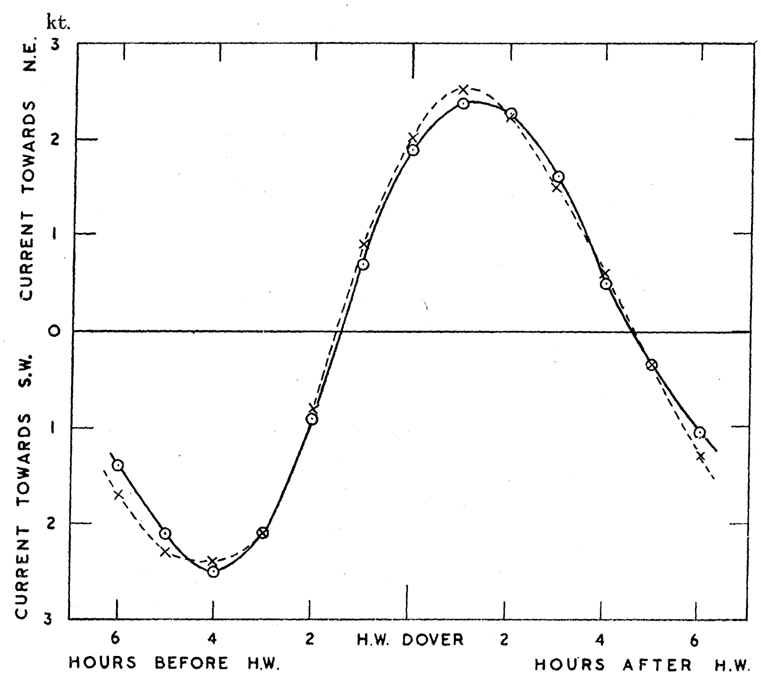

Figure 6. Tidal current across a section from St Margaret's Bay to Sangatte; showing spring rate of mean current over section. —⊙— from *Admiralty tables of tidal streams*. —×— from cable measurements, with conversion factor 1 V = 2·75 knots.

this ratio to be the same for currents as for tidal elevations, and were then plotted, as shown in figure 6, the curve obtained being compared with a curve of the mean tidal current across the section, derived as follows. From *Tidal streams of European waters*, part II (Admiralty 1948), stations V 10, 11, 12 and 13 were selected, as being on a line running parallel to the cable and about 5 km to the north-east of it and therefore giving a reasonable representation of the current across the cable. The component of current in direction 042½° true, i.e. perpendicular to the St Margaret's Bay–Sangatte section, was tabulated for each hour before and after high water at Dover for each station. A mean value for each hour was taken, giving the values for the two central stations, V 11 and 12, twice the weight of values for stations V 10 and 13, in the shallower water towards the sides. The mean values were multiplied by the factor 0·87 to convert from the mean rate from surface to 5 fm, as tabulated, to a mean from surface to bottom at the mean depth of the section. This factor is consistent with the results of Van Veen (1938), based on vertical profiles of tidal currents measured at various positions in the Straits.

Figure 6 shows the two curves obtained in this way, that derived from the cable measurements actually being based on the mean of two successive 29-day periods of observations. The amplitude of the curve from the voltage measurements has been adjusted to that of the other, but the agreement in times of maximum flood and maximum ebb flow and in the general shape of the curves is evidence that the cable voltage does represent the mean flow of water across the section.

The 58 days' observations on which figure 6 is based lead to a calibration factor of 1 V = 141 cm/s (2·75 knots), but this factor depends on the conductivity of the sea water, which is a function of its temperature and salinity and hence varies with season. A separate value of the calibration factor has been derived for each month, but, instead of using the hourly values, a mean value of the ratio of range of current to range of tidal elevation has been computed for the month.

If R_V = range of velocity, R_H = range of elevation, R_E = range of voltage, then the required calibration factor f is given by

$$f = \frac{R_V}{R_E} = \frac{R_V}{R_H}\frac{R_H}{R_E}.$$

The values of R_V/R_H was taken as being constant and equal to the ratio for mean spring tides, i.e. $R_V = 4\cdot9$ knots (252 cm/s), $R_H = 18\cdot6$ ft. (567 cm). The mean value of R_H/R_E was found from the values for each day of the month, taking the predicted values of R_H from *Admiralty tide tables* and the observed values of R_E from the cable records.

In the notation of § 2,

$$f = \frac{V}{e} = \frac{V}{E\delta}, \tag{3}$$

where $\delta = e/E$ and is given by equation (2), from which it is seen that

$$\frac{\kappa_0}{\kappa_1} = \frac{2D}{L}\left(\frac{1}{\delta}-1\right). \tag{4}$$

For the St Margaret's Bay–Sangatte section, $Z = 0\cdot43$ gauss, $L = 34\cdot2$ km. Hence $E/V = 14\cdot7$ mV/cm/s. Since the mean depth of the section, at mean tide level, is 35·3 m, the minor axis D of the equivalent semi-ellipse is 44·9 m, and $2D/L = 2\cdot63\times10^{-3}$. The conductivity κ_1 of the sea water was found from the temperature and salinity, monthly mean values of which were obtained from observations at the *Varne* light-vessel and on the Folkestone–Boulogne steamer route, provided by the Ministry of Agriculture and Fisheries. It is possible, therefore, to use equation (4) to compute the effective specific conductivity κ_0 of the sea bed from the calibration factor, determined empirically.

Table 1 gives the calibration factor for each month, from March 1953 to July 1954, the corresponding value of δ, the specific conductivity κ_1 of the sea water, and the effective specific conductivity κ_0 of the sea bed, deduced using equation (4). It is seen that δ varies in a similar way to κ_1, while κ_0 varies less regularly. Since κ_0 depends on the mean conductivity throughout a considerable thickness of rocks below the sea bed, it would not be expected to have much seasonal variation. The mean value of κ_0 from all the values is $10{\cdot}0 \times 10^{-5}$ (ohm cm)$^{-1}$. This may be compared with the values deduced by Longuet-Higgins (1949) who found $3{\cdot}4 \times 10^{-5}$ for the same area, from only 14 h observations, and values of 5·2 and $7{\cdot}8 \times 10^{-5}$ for other parts of the English Channel.

Table 1. Calibration factors and specific conductivities for the St Margaret's Bay–Sangatte section

month	f (mV/cm/s)	δ	$10^3\kappa_1$ (ohm cm)$^{-1}$	$10^5\kappa_0$ (ohm cm)$^{-1}$
1953: Mar.	6·3	0·43	33·1	11·6
Apr.	6·9	0·47	36·2	10·7
May	7·3	0·50	38·5	10·1
June	7·8	0·53	40·2	9·4
July	8·0	0·55	42·4	9·1
Aug.	8·0	0·55	44·4	9·5
Sept.	7·9	0·54	44·4	9·9
Oct.	7·9	0·54	43·0	9·6
Nov.	7·9	0·54	40·8	9·1
Dec.	7·8	0·53	39·9	9·4
1954: Jan.	7·1	0·48	36·4	10·3
Feb.	7·0	0·48	34·6	9·8
Mar.	7·2	0·49	35·3	9·6
Apr.	7·2	0·49	36·6	10·0
May	7·1	0·48	38·4	10·9
June	7·5	0·51	40·4	10·2
July	7·7	0·52	42·7	10·3

The Aldeburgh–Domburg observations were treated in a similar way for the 29-day period from 28 November to 26 December 1953, and compared with the mean tidal current across the section, derived from the data tabulated in *Tidal streams*, part II, for the stations A 00, 17, 19, 21, 22, 24, 43 and 50. The resulting calibration factor was 1 V = 111 cm/s (2·15 knots). For this section the width $L = 145$ km, the minor axis D of the equivalent semi-ellipse is 39·0 m and $2D/L = 5{\cdot}4 \times 10^{-4}$. Taking the value given for the calibration factor, the corresponding value of δ is 0·145, and the ratio of the conductivity of the sea bed to that of the water, κ_0/κ_1, is $3{\cdot}2 \times 10^{-3}$, compared with $2{\cdot}0 \times 10^{-3}$ for the St Margaret's Bay–Sangatte section for the same month. For the average temperature and salinity in the section for that month, $\kappa_1 = 38{\cdot}5 \times 10^{-3}$ (ohm cm)$^{-1}$, so that $\kappa_0 = 12{\cdot}3 \times 10^{-5}$ (ohm cm)$^{-1}$. Longuet-Higgins (1949) deduced a value of $7{\cdot}2 \times 10^{-5}$ (ohm cm)$^{-1}$, from measurements by Cherry & Stovold (1946) covering 48 h, for the same section.

It is seen from the records (figure 4) that there is an appreciable tidal variation of p.d. on the St Margaret's Bay–Canterbury cable, the potential gradient being in the opposite direction to that on the St Margaret's Bay–Sangatte cable. A comparison of ten successive ranges of p.d. on the two cables, between 20 March and 23 March 1953, showed that the ratio of amplitude on the Canterbury cable to that on the Sangatte cable was 0·30. This is in fair agreement with Longuet-Higgins's theoretical treatment (1949). From figure 4 of his paper, the amplitude of the p.d. between the side of a channel and a point inland by a distance 0·75 of the width of the channel, should be about 0·4 of the amplitude across the channel.

For the two 29-day periods of observations from the St Margaret's Bay–Sangatte cable, referred to above, harmonic analyses were carried out by the method given in *Admiralty tide tables*, part III, for the tidal constituents M_2, S_2, N_2, K_1, O_1, M_4 and MS_4, and the amplitudes and phase lags obtained are shown in table 2. Harmonic analyses of currents at the *Varne* light-vessel were made for three fortnightly sets of observations by the Jacobsen current-meter, obtained in 1911, 1912 and 1913 respectively. Doodson (1930) carried out analyses of longer series of observations made in the same way in 1922–3. From his published results, the constants for the mean current from surface to bottom have been computed, expressed in elliptical form, and the amplitude and phase lag of the component along the major axis are given in table 2.

TABLE 2. HARMONIC CONSTANTS OF TIDAL CURRENTS IN THE STRAITS OF DOVER

(Amplitudes in cm/s, phases in degrees)

	constituents							
	M_2	S_2	N_2	K_1	O_1	P_1	M_4	MS_4
from cable measurements:								
amp. V	89	31	16	8	9	—	9	8
phase v	3	51	339	181	59	—	283	333
from *Varne* data, 1922/23:								
bearing of major axis	033	035	—	053	039	059	—	053
amp. W	75·1	24·3	—	7·5	8·3	3·9	—	3·3
phase w	347	35	—	207	50	183	—	211
ratio of amplitudes V/W	1·19	1·27	—	1·07	1·08	—	—	2·4
diff. of phases $v-w$	16	16	—	−26	9	—	—	122

The direction of the major axis is seen to be approximately normal to the direction of the cable, so that this component may reasonably be compared with the cable measurements. The agreement between the two sets of constants is quite good for the semi-diurnal constituents. The amplitude is somewhat greater and the phase later at the cable section, compared with the *Varne*, but these effects are probably genuine due to the difference in position. On the other hand, the differences in amplitude may arise from uncertainty in the method of calibration of the cable measurements by the tables of tidal streams. The agreement in the amplitude of the diurnal components is fairly good, but the difference in phase lags between K_1 and O_1 is greater than would be expected. It is possible that the constants for K_1 from the cable measurements may be affected by a constituent of the solar diurnal period occurring in the earth-current system and not associated with the flow of water. No inference can be drawn from the quarter diurnal constituents, as the individual results are so variable. (For example, the *Varne* data for 1922 gave MS_4 an amplitude of 5·1 cm/s and phase 237°, while for 1923 they gave 1·4 cm/s and phase 185°.)

5·2. *Residual currents*

As an approximate method of eliminating the tidal constituents of the current, the mean of 25 successive hourly values, centred at noon, was computed for each day of the observations. These 25 h means of current provide the basis of much of the further analysis given in following sections. When considering the correlation between current and differences of sea level, 25 h means were deduced from measurements of the tide-gauge records in a similar way.

For several selected periods, during which unusually large currents were recorded, a more detailed analysis was carried out, eliminating the tidal constituents by the method devised by Doodson (1929) for the analysis of storm surges. This method yields values of the residual current at 3-hourly intervals.

It was not found possible to obtain an absolute measurement of the mean current by the cable method, owing to the presence of a constant potential difference of approximately 0·3 V (St Margaret's Bay positive with respect to Sangatte), which may be due partly to differences in electro-chemical potentials and partly to permanent earth-current systems. The residual currents which are the subject of this analysis are, therefore, variations of current from a long-period average. Carruthers (1930) found that the mean current over a long period averaged 2·7 miles/day (5·8 cm/s) towards the north-east.

5·3. *Subsidiary data*

The wind recorded at Lympne airport, situated 12 miles (19 km) to the west of Dover at a height of 340 ft. (104 m) above sea level, was taken as representative of the local wind in the Straits of Dover. Tables of hourly mean values of speed and direction, obtained from Dines anemometer records, were provided by the Meteorological Office. From these, the component along the channel of the wind velocity, W, or of its square, W^2, was computed. It is probably sufficiently accurate to assume that the stress of the wind on the sea surface is in the direction of the wind and is proportional to the square of the wind speed, at least for the higher wind speeds. Thus the component of wind stress in the direction of the current should be proportional to $W^2 \cos\phi$, where ϕ is the angle between the direction towards which the wind is blowing and the normal to the section, which may be taken with sufficient accuracy as towards the north-east (true bearing 045°). The 25 h means of current have, therefore, been correlated with the corresponding means of $W^2 \cos\phi$, and in the same way the 3-hourly values of current have been correlated with 3-hourly means of $W^2 \cos\phi$.

An investigation, described in § 6·5, was made of the correlation between the residual current through the Straits and the wind over the southern North Sea or over the English Channel as computed from the barometric pressure, following the methods of Dietrich & Wyrtki (1952). For the southern North Sea, the same three stations, i.e. Tynemouth, Paris and Emden, were taken, but for the English Channel the three stations Scilly Isles, Rennes and Felixstowe were selected. In each case, the pressure at 00, 06, 12 and 18 h was taken from data which were made accessible by the Meteorological Office, and the mean pressure for the day, centred at noon, derived, giving the observations at 00 and 24 h half the weight of those at 06, 12 and 18 h. From the daily mean pressure at the three points of each triangle, the mean surface wind over the region was computed, using a factor of 0·68 for the North Sea and 0·67 for the English Channel for converting the geostrophic wind speed to speed at the surface and allowing for an angle of 14° between the surface wind and the normal to the direction of greatest pressure gradient, as assumed by Dietrich and Wyrtki.

For the comparison of the current with differences in sea level, tide-gauge records for

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varying periods between March 1953 and March 1954 were kindly lent by the authorities concerned for the ports of Southend, Dover, Shoreham, Newhaven, Portsmouth and Devonport on the English side and for the continental ports of Flushing, Dieppe, St Malo and Brest. In addition, hourly tabulations of tidal heights and differences between the observed and predicted values for Lowestoft were provided by the Hydrographic Department of the Admiralty. From the tide-gauge records, hourly heights were read off for the periods required and 25 h means computed or the more detailed analysis applied, as in the case of the currents.

Much of the analysis involved computing correlations between the current, wind components and differences of sea level and setting up regression equations between them. The definitions and formulae used in these computations follow general practice in statistics (e.g. Yule & Kendall 1947).

6. Results of analysis

6·1. *Daily values of current*

The 25 h mean values of the current across the St Margaret's Bay to Sangatte section, centred at noon each day, are shown in figures 7, 8, 9 and 10 for the period 25 February 1953 to 31 March 1954.

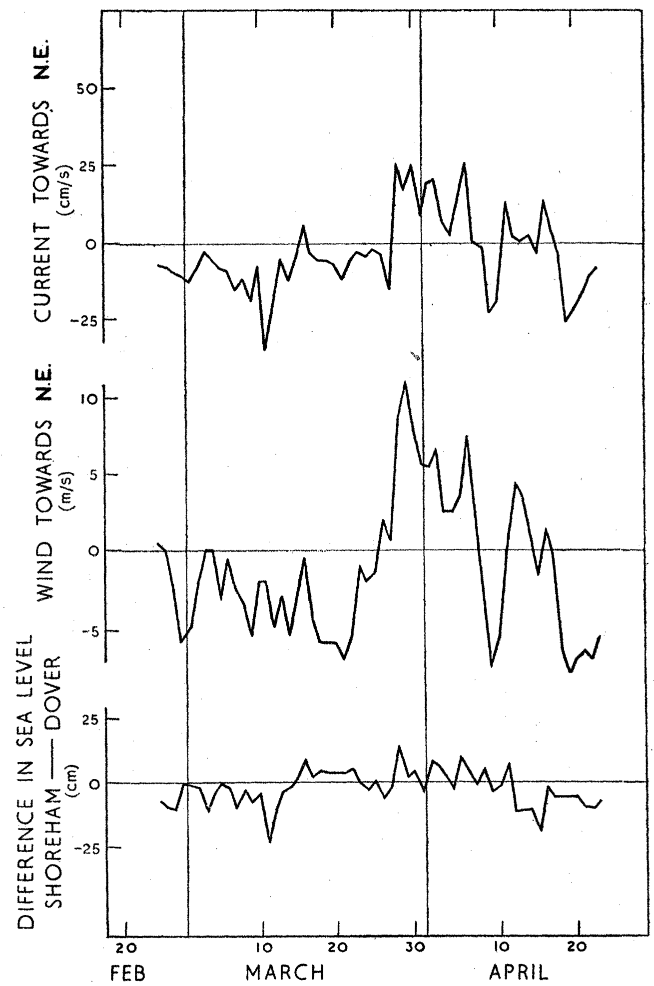

Figure 7. 25 h means of current, wind component and difference in sea level, February to April 1953.

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For the period 25 February to 31 August 1953, the component towards the north-east of the wind recorded at Lympne, as derived from the values at 6-hourly intervals given in the Daily Weather Report, is shown in the figures. The correlation between the current and this component of wind is evident. During the summer months, May to August 1953, the rates of flow were comparatively small. A more detailed analysis was carried out for the seven months 1 September 1953 to 31 March 1954, and for this period the figures show the 25 h means of $W^2 \cos \phi$, the component of W^2 towards the north-east, obtained from the hourly tabulations of wind at Lympne. The figures also show the corresponding values of

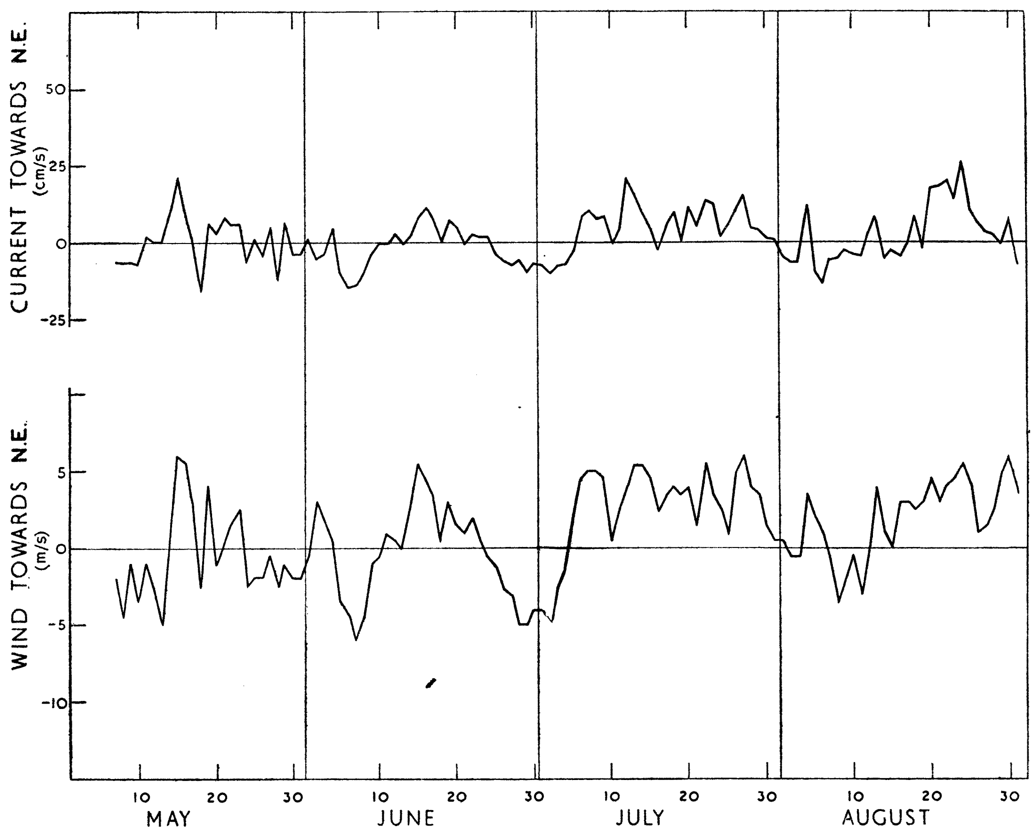

FIGURE 8. 25 h means of current and wind component. May to August 1953.

$\Delta\zeta$, the difference between the sea level in the English Channel, as given by the mean of values at Shoreham and Dieppe, and in the southern North Sea, as given by the mean of values at Lowestoft and Flushing.

It is seen from figures 9 and 10 that a peak of unusually strong flow generally coincides with a peak in the $W^2 \cos \phi$ or $\Delta\zeta$ curve, or in both. The strong north-easterly flow on 21 September 1953 was associated with a large value of $W^2 \cos \phi$ and a moderate value of $\Delta\zeta$. On 1 November, however, a stronger north-easterly current occurred with a lower value of $W^2 \cos \phi$ and a high $\Delta\zeta$, indicating that the slope of the surface was the main factor in causing the strong flow on this occasion. The same is true of the strong south-westerly flow on 3 January 1954, which corresponded to a large negative value of $\Delta\zeta$, but no peak in the $W^2 \cos \phi$ curve. On the other hand, a less intense but more sustained south-westerly flow occurred between 29 January and 5 February 1954, associated with large negative values of $W^2 \cos\phi$ (corresponding to persistent north-easterly winds over the North Sea and English Channel), although the $\Delta\zeta$ values were small.

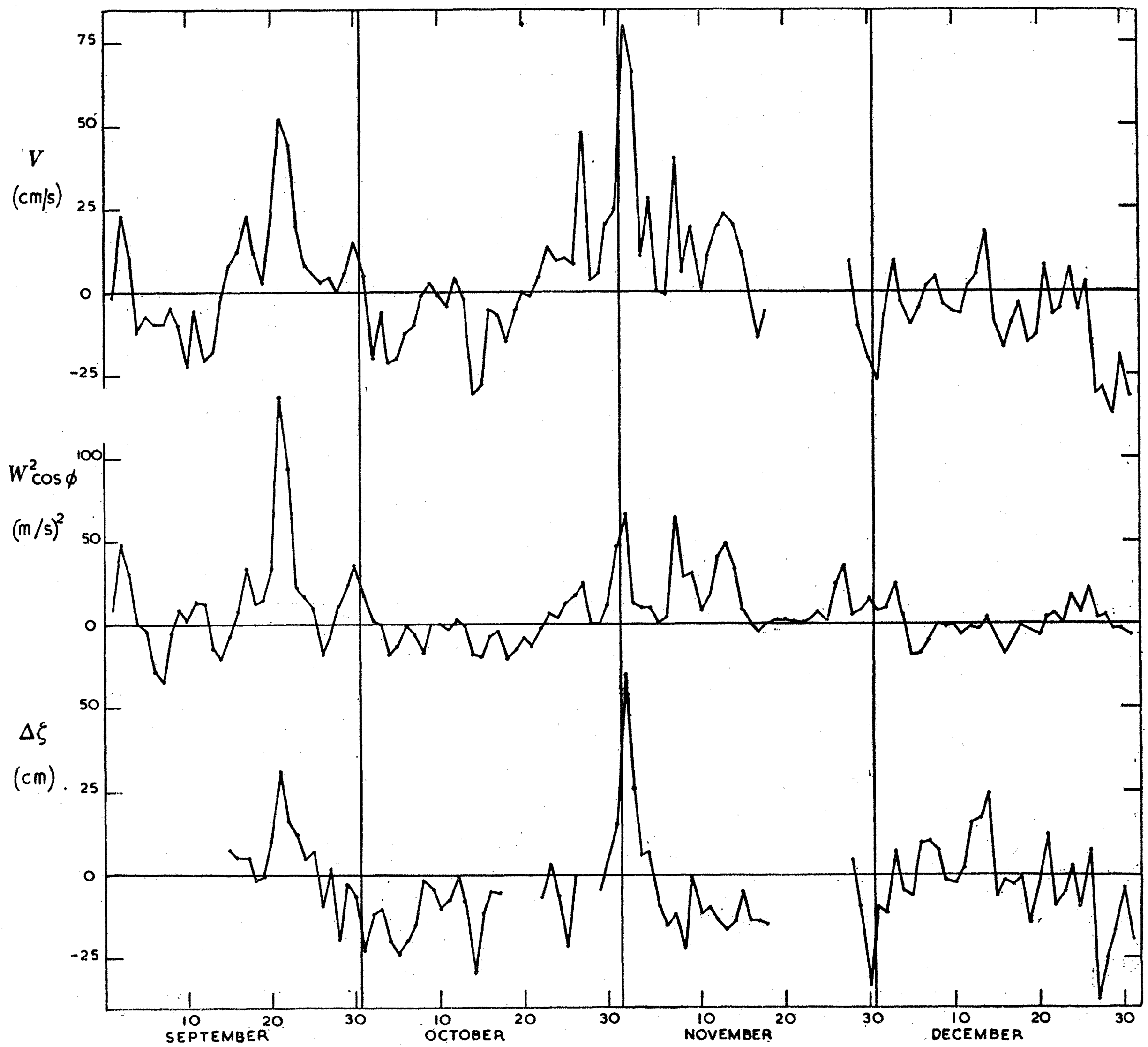

FIGURE 9. 25 h means of V, $W^2 \cos\phi$ and $\Delta\zeta$, September to December 1953. V = current towards north-east, $W^2 \cos\phi$ = wind function, $\Delta\zeta$ = difference in sea level: mean of Shoreham and Dieppe – mean of Lowestoft and Flushing. W = wind speed at Lympne.

6·2. *Comparison of cable results with vertical log data*

During the period covered by the records of cable voltage, current measurements by the Carruthers vertical log were being carried out from the *Varne* light-vessel, situated in mid-channel, about 20 km to the south-west of the St Margaret's Bay–Sangatte line. The design and operation of the vertical log current meter have been described by Carruthers (1935 *b*, 1938). In the measurements made at the *Varne* since World War II, the cup system has been at an average depth of 4 m until the end of 1953, afterwards 6 m, and the counters have been read at intervals of a mean lunar day of 24 h 50 min. In this way the residual current for successive lunar days has been obtained, and Dr Carruthers and Cdr Lawford have kindly made the data available for comparison with the cable results. The comparison between the two sets of results appeared fairly good from a qualitative point of view, but, since the periods for which the residual currents were deduced were different, a quantitative comparison was not possible.

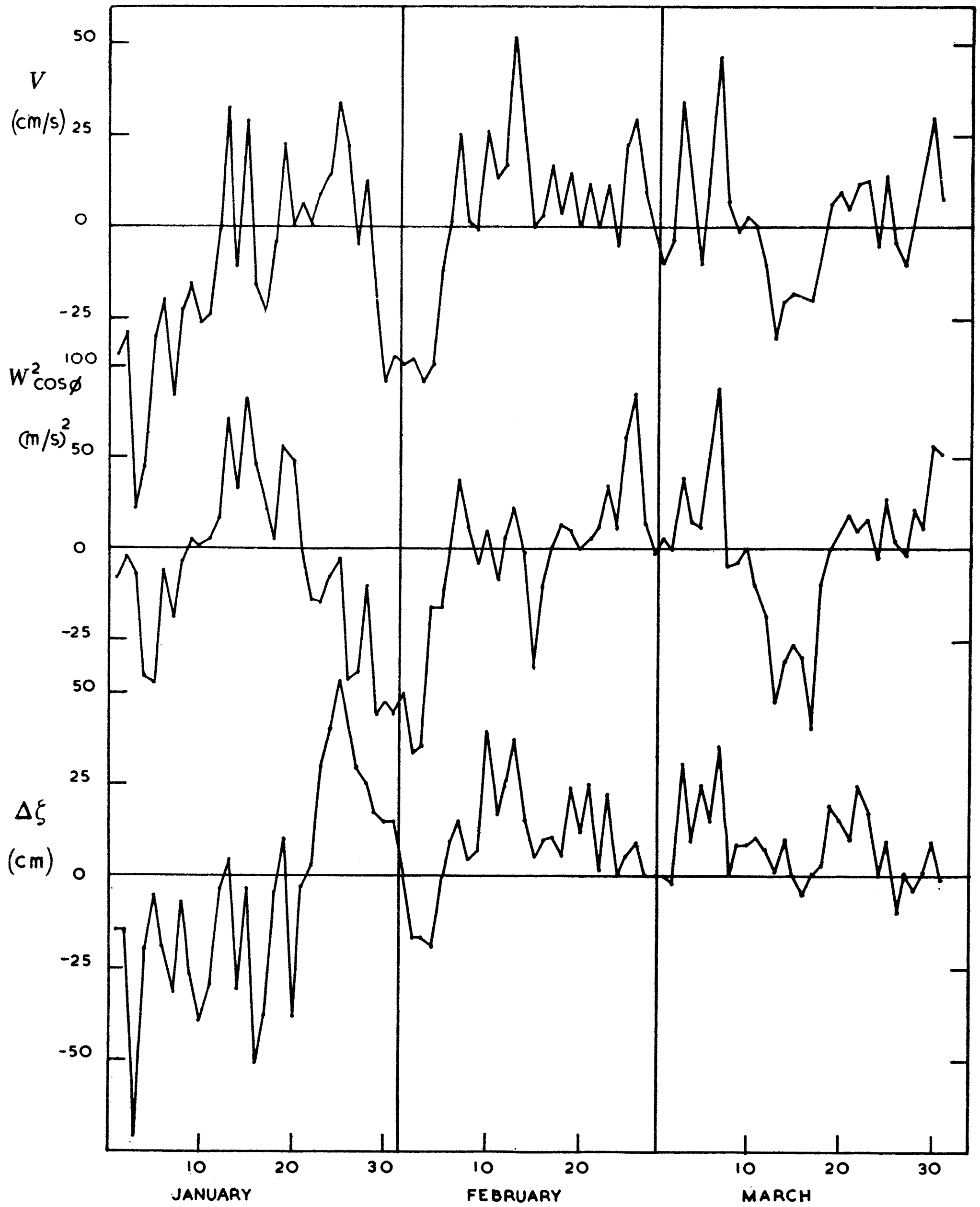

FIGURE 10. 25 h means of V, $W^2 \cos \phi$ and $\Delta\zeta$, January to March 1954 (definitions as for figure 9).

During May and June 1954, however, trials of an alternative method of recording vertical log measurements were carried out at the *Varne*, the direction of flow and the revolution counter reading being noted every hour. It was possible, therefore, to compute 25 h means, centred at noon, of the current component in a direction N 50° E (magnetic), i.e. normal to the cable section, in the same way as from the cable measurements. A comparison of the results for May is shown in figure 11. A similar comparison could not be made for June, as there were several gaps in the cable records due to the cable being required for telephone service. The correspondence between the two sets of means is quite good, the vertical log curve being displaced relative to the cable one by an amount equivalent to an average current of 8 cm/s towards the north-east for the month. The basic current for May, derived from the vertical log data, was 5 cm/s (2·4 miles/day) towards the north-east. The cable results, however, do not give the absolute current, but only the current relative to a long-period mean. Considering that the vertical log data represent the current in one position and at one depth whereas the cable measurements give the mean current over a whole section, the agreement between the results by the two methods, and the mutual confirmation which they provide, may be regarded as very satisfactory.

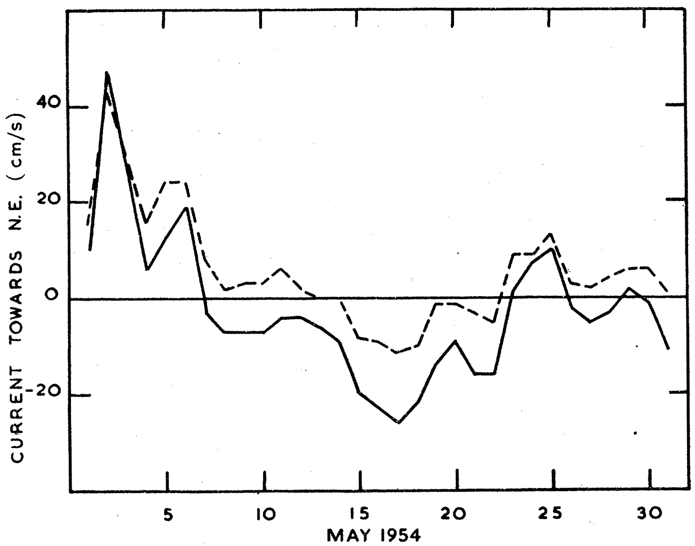

FIGURE 11. Comparison of current as determined by cable measurements and by vertical log during May 1954. Full line: St Margaret's Bay–Sangatte cable. Broken line: vertical log at *Varne* light-vessel.

6·3. *Comparison of transport across the St Margaret's Bay–Sangatte and Aldeburgh–Domburg sections*

For the period 28 November 1953 to 31 March 1954, the 25 h means, centred at noon, of the transport across the St Margaret's Bay–Sangatte and Aldeburgh–Domburg sections have been computed, the cross-sectional areas of the two sections being 1·20 and 4·44 km^2 respectively. The results are shown in figure 12. The degree of correspondence between the two transports is an indication of the extent to which the flow through the Straits of Dover is part of a general flow through the southern part of the North Sea. The coefficient of correlation between the currents across the two sections is 0·86.

A difference between the transports, if genuine, implies an accumulation in, or loss of water from, the region between the two sections, with a rise or fall in the mean sea level. Since the surface area between the sections is 13,870 km^2, an excess of 1 km^3/day of water flowing in through the St Margaret's Bay–Sangatte section over that flowing out through the Aldeburgh–Domburg section would correspond to an increase of 7·2 cm in mean sea level during the day. An attempt was made to relate the difference in transport to the changes in mean sea level of the region, as deduced from the values of sea level at Lowestoft, Flushing and Dover, but without success, as the accuracy of the transports measured across the Aldeburgh–Domburg section was not high enough for the purpose. Owing to the greater cross-sectional area, the currents across the section are less than across the St Margaret's Bay–Sangatte section, while the fluctuations of the record due to magnetic disturbances are greater, as discussed in § 4.

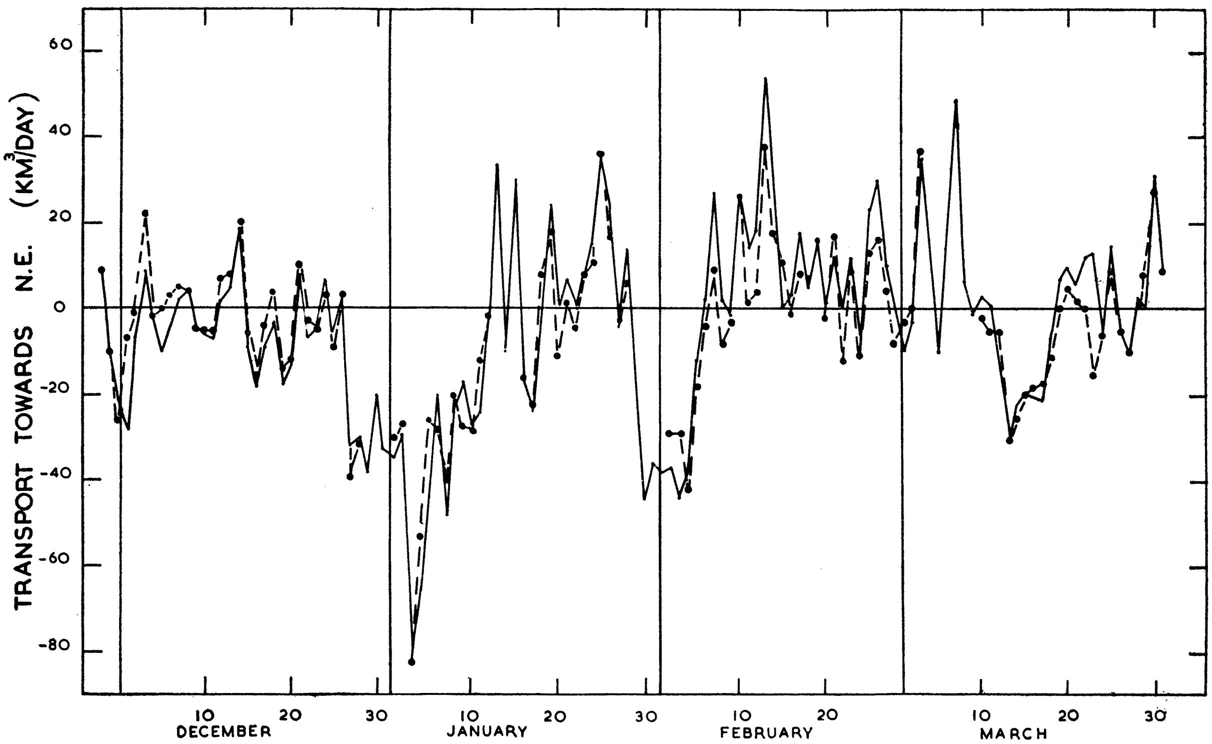

FIGURE 12. Comparison of transports of water across St Margaret's Bay–Sangatte and Aldeburgh–Domburg section, 28 November 1953 to 31 March 1954. —— St Margaret's Bay–Sangatte. —·— —·— Aldeburgh–Domburg.

6·4. *Correlation of current with wind and surface gradient*

The forces which directly influence the movement of water through the Straits of Dover are the tangential stress of the wind on the sea surface and the horizontal pressure gradient due to differences in level of the water. The effect of a gradient in barometric pressure along the direction of the Straits is also to be considered, but it is found to be small compared with that due to differences of sea level, and may be neglected to a first approximation. The 25 h mean values have been represented, therefore, by an equation of the form

$$V = aW^2 \cos\phi + b\Delta\zeta, \tag{5}$$

where V denotes the mean current across the St Margaret's Bay–Sangatte section, as determined by the cable measurements. W denotes the wind speed at Lympne and ϕ the angle between the direction of the wind vector and the normal to the section. $\Delta\zeta$ denotes the difference in sea level between the eastern part of the English Channel, as given by the mean of values at Shoreham and Dieppe, and the southern part of the North Sea, as given by the mean of values at Lowestoft and Flushing. V is in cm/s, W in m/s, and $\Delta\zeta$ in cm. A dynamical basis for a relation of this form is discussed in § 7·1.

The values of the regression coefficients a and b have been determined from the analysis of the data. The resulting values of a and b as obtained by analyzing the observations in two groups, from 15 September to 31 December 1953 and from 1 January to 31 March 1954, and also as a single group, are given in table 3.

TABLE 3. VALUES OF COEFFICIENTS IN EQUATION (5)

period	a	b	R
15 Sept. to 31 Dec. 1953	$0{\cdot}44 \pm 0{\cdot}07$	$0{\cdot}65 \pm 0{\cdot}11$	0·83
1 Jan. to 31 Mar. 1954	$0{\cdot}33 \pm 0{\cdot}03$	$0{\cdot}73 \pm 0{\cdot}09$	0·92
15 Sept. 1953 to 31 Mar. 1954	$0{\cdot}37 \pm 0{\cdot}03$	$0{\cdot}70 \pm 0{\cdot}07$	0·88

The standard errors of the coefficients and the total correlation coefficient are shown in each case. The differences between the coefficients derived from the two groups do not appear to be significant. From the data treated as a whole,

$$V = 0{\cdot}37W^2\cos\phi + 0{\cdot}70\Delta\zeta, \tag{6}$$

where V is in cm/s, W in m/s and $\Delta\zeta$ in cm. The total correlation coefficient, 0·88, is high, implying that the greater part of the observed flow may be attributed to the effects of wind and surface gradient, as has been assumed. Other relations, with different definitions of $\Delta\zeta$, e.g. taking $\Delta\zeta$ as the difference between levels at Shoreham and Dover, were tried, but the total correlation coefficients were smaller and so these alternative methods were not pursued.

A possible interpretation of the coefficients a and b in terms of the coefficients of wind stress and bottom friction is discussed in § 7·1.

6·5. *Correlation of current with regional winds*

While the forces directly influencing the current through the Straits are the local wind stress and the differences of sea level, such differences of level are also due, very largely, to the effects of wind over larger areas of water. As an alternative method of presentation, therefore, the daily values of current V have been correlated with the regional wind over the southern part of the North Sea or the English Channel. This follows the method used by Wyrtki (1952*a*) in his study of monthly means of flow through the Straits of Dover, based on 3-day averages obtained by the Carruthers drift indicator. Wyrtki (1952*b*) has also investigated the changes in mean sea level of the North Sea from month to month and the inflow of water through the Straits of Dover and from the north, as related to regional winds. In the present investigation, the winds were those deduced from the daily mean barometric pressures, as described in § 5·3.

A relation of the form

$$V = cW_N + dW_E \tag{7}$$

was assumed, where W_N and W_E are the north-going and east-going components of the wind velocity. When the regression coefficients c and d have been computed, the equation may be put in the form

$$V = AW\cos(\theta - \alpha), \tag{8}$$

where $A^2 = c^2+d^2$ and $\tan\alpha = d/c$, W being the resultant wind velocity, blowing towards a direction making an angle θ to the east of north. α represents the direction of the wind vector which is most effectively associated with north-east-going flow through the Straits. This method of representation was used by Doodson (1924) in investigating the influence of wind on perturbations of sea level.

For the 7-month period from 1 September 1953 to 31 March 1954, V was correlated with winds over both the southern North Sea and the English Channel, and the values of the coefficients with their standard errors and the total correlation coefficient R were as follows:

	A	α	R
winds over southern North Sea	$3{\cdot}6 \pm 0{\cdot}35$	16°	0·83
winds over English Channel	$2{\cdot}85 \pm 0{\cdot}35$	30°	0·76

V was expressed in cm/s and W in m/s, as before. The similarity of the results arises from the high correlation between the regional winds over the two areas, the coefficient of correlation between the northerly components being 0·75 and between the easterly components 0·89.

The further analysis by this method was confined to winds over the southern North Sea. When the data for the previous six months, 1 March to 31 August 1953, were investigated, a significantly lower value of A was obtained, as shown in table 4, which also gives the mean values for the whole 13-month period.

Table 4. Values of coefficients in equation (8)

period	A	α	R
1 Mar. to 31 Aug. 1953	$2{\cdot}25 \pm 0{\cdot}35$	39°	0·77
1 Sept. to 31 Mar. 1954	$3{\cdot}6 \pm 0{\cdot}35$	16°	0·83
1 Mar. 1953 to 31 Mar. 1954	$3{\cdot}1 \pm 0{\cdot}25$	21°	0·79

Table 5. Variation of wind factor A throughout the year

period		A	$(\overline{W^2})^{\frac{1}{2}}$ (m/s)
1953:	Mar./Apr.	$3{\cdot}4 \pm 0{\cdot}8$	6·3
	May/June	$1{\cdot}5 \pm 0{\cdot}35$	5·3
	July/Aug.	$2{\cdot}0 \pm 0{\cdot}5$	5·7
	Sept./Oct.	$3{\cdot}0 \pm 0{\cdot}5$	6·3
	Nov./Dec.	$4{\cdot}0 \pm 0{\cdot}9$	7·0
1954:	Jan./Feb.	$4{\cdot}0 \pm 0{\cdot}7$	8·8
	1 Mar./14 Apr.	$2{\cdot}9 \pm 0{\cdot}6$	6·6
	15 Apr./9 June	$2{\cdot}5 \pm 0{\cdot}6$	6·0
	19 June/31 July	$2{\cdot}7 \pm 0{\cdot}7$	6·6

Since the value of the coefficient A appeared significantly different for the two periods, the analysis was carried out for 2-monthly periods from 1 March 1953 to 31 July 1954. The resulting values and their standard errors are shown in table 5. It appears that A is significantly greater in the winter months. This may be due partly to the greater wind speeds in these months. A linear relation between V and W has been assumed, but if V is proportional to W^2, which is more probable, the computed values of A should be proportional to W. The third column of table 5 gives the values of $(\overline{W^2})^{\frac{1}{2}}$ for the same periods, and the changes in A are seen to correspond with them to some extent, although the variation in $(\overline{W^2})^{\frac{1}{2}}$ does not appear large enough to account for the whole variation in A.

Another possible effect is that, for a given geostrophic wind speed, the wind speed at the sea surface, and so the stress, may be greater when the air in the lower atmosphere is in an unstable condition, as it is more frequently during the winter months.

The mean value of the coefficient A, which has been termed the wind factor by previous investigators, i.e. $A = 3\cdot1$ when V is in cm/s and W in m/s, may be compared with the value 1·6 obtained by Wyrtki (1952*a*) by considering monthly mean values of the current at the *Varne* light-vessel and the wind over the southern North Sea. Carruthers, Lawford & Veley (1950), from an analysis of the vertical log measurements at the *Varne* from March 1938 to November 1939, taking the residual current over periods of a lunar day, and comparing it with the wind velocity as estimated at the light-vessel itself, found a factor of 1·5. From the vertical log data obtained in 1951 to 1953, Lawford (unpublished) has deduced a mean value of 2·0 from the monthly means and 1·86 from the daily observations. The tests referred to in § 6·2 indicated that the new method of recording the vertical log measurements would give measured velocities about 20 % greater, with a corresponding increase in the wind factor derived from them.

The total correlation coefficient for the method of presentation in terms of regional winds, although fairly high at 0·79, is rather less than the corresponding value of 0·88 for equation (6) of § 6·4. As a link between the two methods, the correlation between the difference in sea level $\Delta\zeta$ and the wind components W_N and W_E was computed, leading to the equation

$$\Delta\zeta = 3\cdot25 W \cos(\theta - 26^\circ),$$

where $\Delta\zeta$ is in cm and W in m/s, the standard error of the coefficient being $\pm 0\cdot4$. The total correlation coefficient is 0·71, a value which indicates that an appreciable part of the variations in $\Delta\zeta$ was due to causes other than the wind over the southern part of the North Sea.

6·6. *Results of detailed analysis for selected periods*

The method of analyzing the current V, wind function $W^2 \cos\phi$ and difference in sea level $\Delta\zeta$ to obtain values at 3-hourly intervals for selected periods has been described in §§ 5·2 and 5·3. The following periods have been treated in this way: (*a*) 19 to 24 September 1953, (*b*) 26 October to 8 November 1953, (*c*) 1 to 6 January 1954. (*a*) and (*b*) were periods of strong flow through the Straits towards the north-east and (*c*) a period of strong flow towards the south-west. These occasions will be described briefly in turn.

(*a*) 19 *to* 24 *September* 1953

In figure 13, the first curve on the left shows the current V, reaching a peak of 85 cm/s at 18.00 h on 21 September. The second curve shows $W^2 \cos\phi$, reaching a peak almost simultaneously with V. The third curve gives $\Delta\zeta$ between a line from Newhaven to Dieppe and a line through Dover, i.e. the downward slope of the sea surface in the eastern end of the Channel. The fourth curve shows $\Delta\zeta$ between the standard sections Newhaven to Dieppe and Lowestoft to Flushing, i.e. sections at approximately equal distances on either side of the Straits.

The right-hand curves show the changes in sea level during the same period at several ports in the English Channel and North Sea. A disturbance appears first at Brest, with a peak at 06.00 h on 21 September, and later at other ports in the English Channel. At Dover, however, the level is unaffected by this surge but later falls, following a fall in level at Lowestoft. The curve for Flushing is again different, presumably due to effects of local winds in the southern North Sea. The charts published in the Daily Weather Report for this period show an intense depression which crossed the centre of the British Isles, the cold front passing the Straits of Dover at 06.00 h on 21 September. Before this time there had been

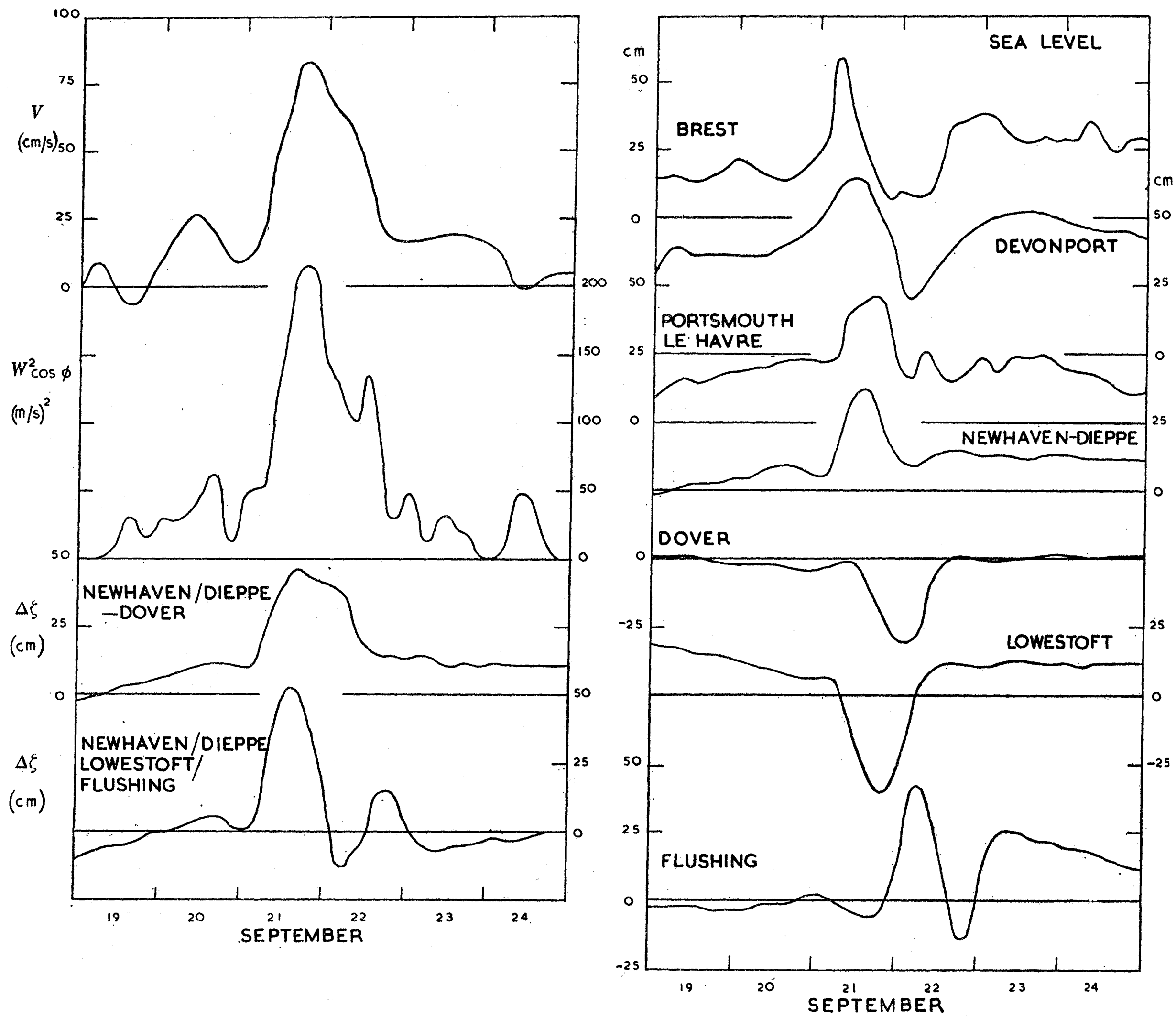

FIGURE 13. 3-hourly values of current V, wind function $W^2 \cos\phi$, difference in sea level $\Delta\zeta$ and sea level ζ at various positions, 19 to 24 September 1953. (Definitions as for figure 9.)

strong westerly winds in the Channel and to the west of it, but comparatively light winds in the North Sea.

This is an example of strong north-east-going flow, associated with a surge which travelled up the Channel but had little effect on the elevations beyond the Straits of Dover.

(*b*) 26 *October* to 8 *November* 1953

The curves for this period are shown in figure 14. A sharp peak of current towards the north-east occurred on 27 October, and a more sustained pulse in the same direction on 1 to 2 November. On neither occasion were the local south-westerly winds, as indicated by the curve of $W^2 \cos\phi$, unusually strong, but on both occasions there are peaks in the $\Delta\zeta$ curve. Whereas the flow on 21 September would appear to have been due more to the local wind stress, on 1 November the slope of the sea surface would seem to have been the main factor producing the flow.

The meteorological conditions on 27 October and 1 November showed similar features, a depression being centred over the north of the British Isles, with south-westerly winds in

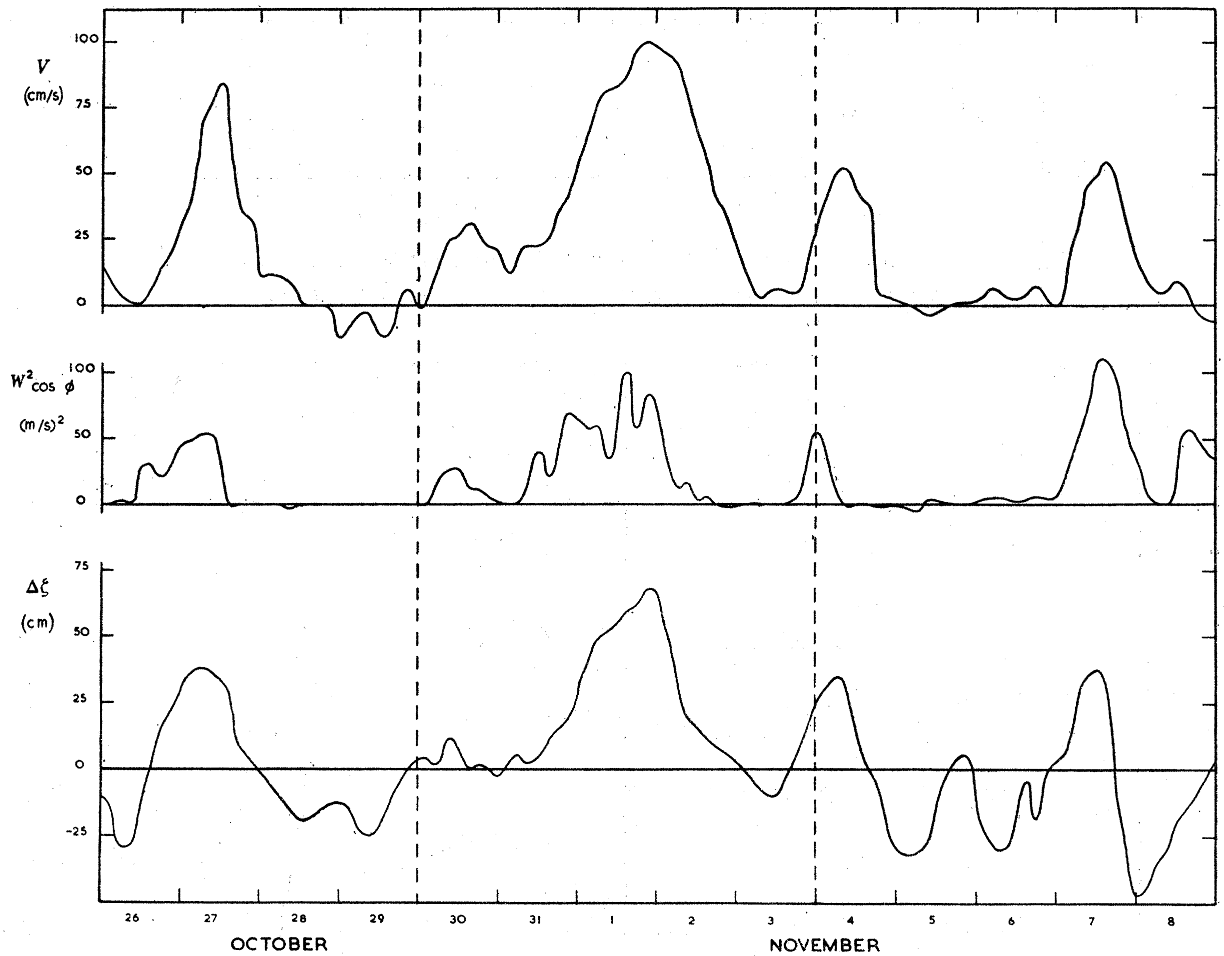

FIGURE 14. 3-hourly values of current V, wind function $W^2 \cos\phi$ and difference in sea level $\Delta\zeta$, 26 October to 8 November 1953.

the Channel and southerly winds over the North Sea. This type of situation was referred to by Carruthers (1930) as being the most favourable for north-east-going flow through the Straits.

Figure 15 is plotted directly from the hourly readings for the period from 30 October to 3 November 1953. It was a period of neap tides and it is seen that the north-east-going current was strong enough to prevent any south-westerly flow on two successive ebb tides.

(*c*) 1 *to* 6 *January* 1954

Figure 16 shows the curves for this period. A strong south-westerly flow is shown by the curve of V, reaching a peak of nearly 100 cm/s at 21.00 h on 3 January. This flow is also shown on the records from the Aldeburgh–Domburg cable (second curve), but naturally the velocity is less, corresponding to the greater area of the section. The $W^2 \cos\phi$ curve indicates no

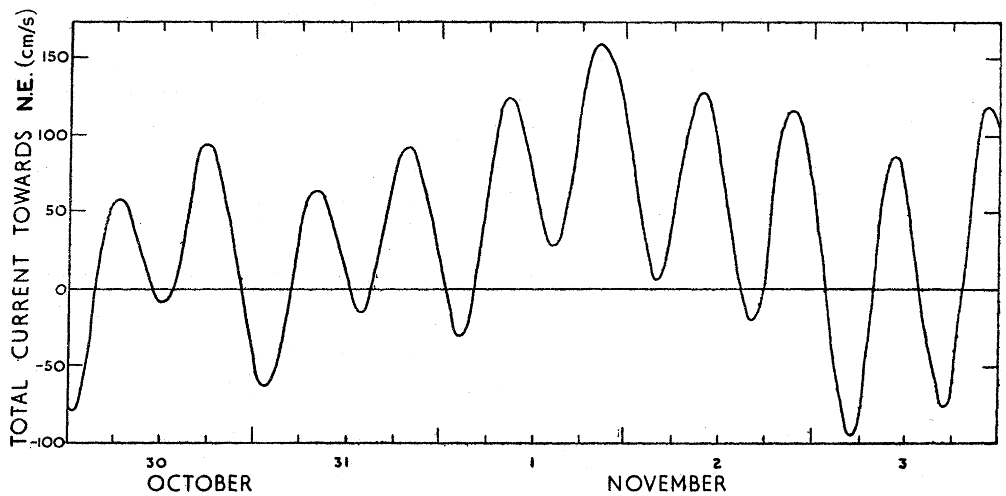

FIGURE 15. Current curves for the period 30 October to 3 November 1953, showing north-east-going drift superimposed on tidal currents.

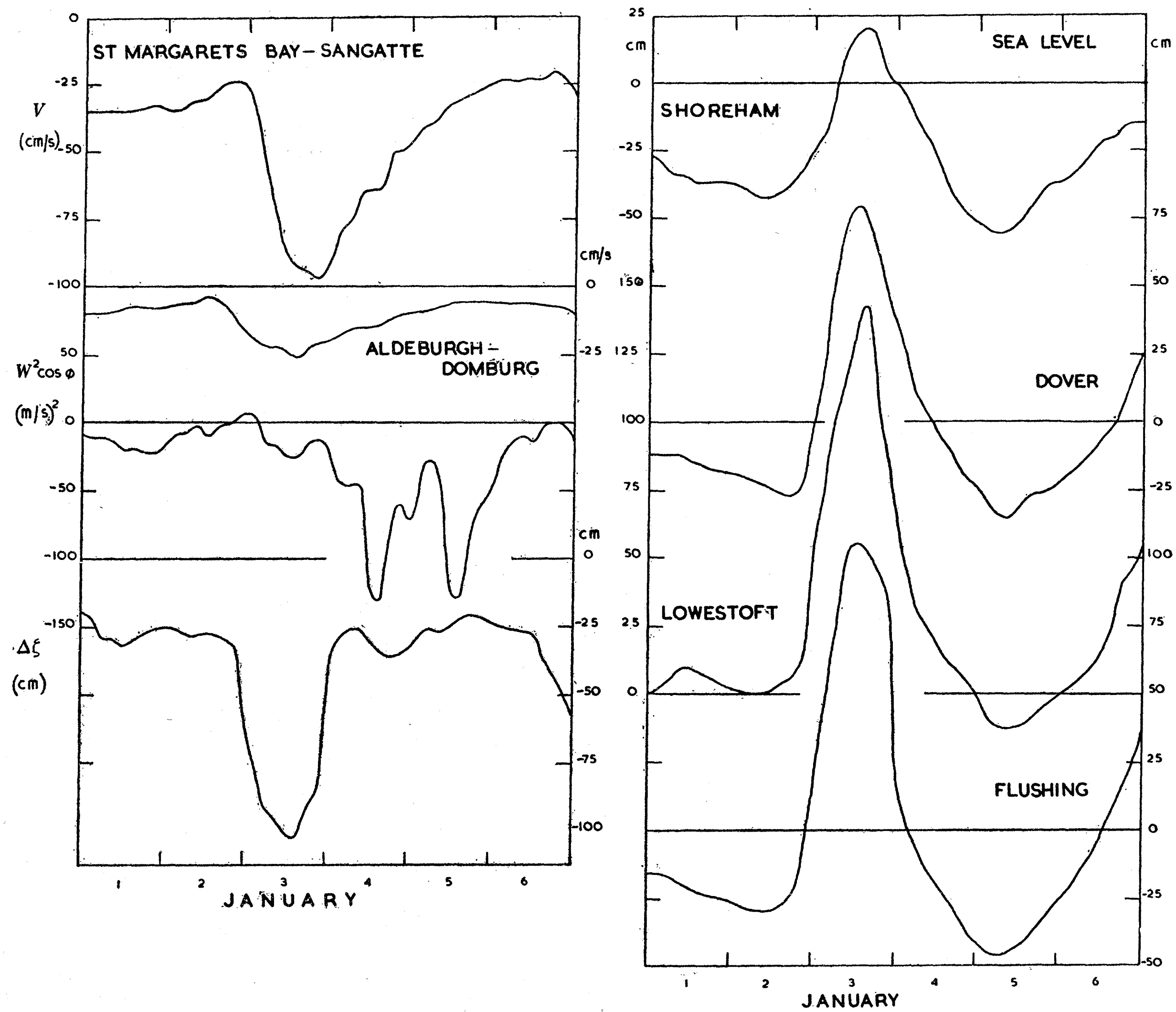

FIGURE 16. 3-hourly values of current V, wind function $W^2 \cos \phi$, difference in sea level $\Delta\zeta$ and sea level ζ at various positions, 1 to 6 January 1954.

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large wind velocities locally until *after* the current surge. The curve of $\Delta\zeta$, on the other hand, shows a large downward slope from the North Sea towards the English Channel, reaching a peak of about 100 cm for the difference between Lowestoft/Flushing and Shoreham shortly before the peak flow.

The sea level curves, on the right, show a raising of the sea level, reaching a peak almost simultaneously at Lowestoft and Flushing in the North Sea, at Dover and at Shoreham. The rise is greatest at Lowestoft, reaching 140 cm, and less at Dover and Shoreham, corresponding to the downward slope shown by the curve of $\Delta\zeta$.

An anticyclone persisted west of Ireland during this period, while a depression developed off the coast of northern Norway and moved south-eastwards. Strong northerly winds acted on the whole of the North Sea during 3 January, affecting the English Channel as well later on that day and during 4 January. The conditions were rather similar, although on a less severe scale, to those prevailing during the surge of 31 January and 1 February 1953.

7. Dynamical interpretation of the results

7·1. *Steady state*

In a previous paper (Bowden 1953) the writer has indicated a method by which the steady-state flow along a channel of variable cross-section may be computed in terms of the surface wind stress and the surface gradients. It is assumed that the wind current is superimposed on a tidal current. This method of calculation has been applied to the eastern part of the English Channel and southern part of the North Sea, between sections running approximately from Shoreham to Dieppe and from Lowestoft to Flushing, treating it as a single channel which contracts and then opens out again.

Rectangular co-ordinate axes are taken, with the origin at the bottom, Ox along the axis of the channel and Oz vertically upwards. The following notation is used:

u component of velocity parallel to Ox,
u_1 amplitude of tidal current, parallel to Ox, at the bottom,
u_0 component parallel to Ox of drift velocity at the bottom,
$\bar{u}$ mean drift velocity from surface to bottom,
ζ elevation of surface above mean sea level,
F_s component of wind stress on the surface, parallel to Ox,
F_0 component of frictional stress at the bottom, parallel to Ox,
h depth of water,
ρ density of water,
ρ_a density of the air,
g acceleration due to gravity,
σ angular velocity of semi-diurnal tidal constituent,
k coefficient of bottom friction,
γ^2 coefficient of wind stress on the surface,
N vertical eddy viscosity, assumed constant with depth,
A cross-sectional area,
B breadth of channel,
L length of channel.

Considering motion along the channel only, if u_b is the instantaneous velocity at the bottom, the frictional stress of the bottom on the water immediately above it is directed along the negative Ox direction and is given by $-F_b$, where

$$F_b = k\rho\,|\,u_b\,|\,u_b. \tag{9}$$

If

$$u_b = u_0 + u_1 \cos \sigma t,$$

it may be shown, by taking a mean over a tidal period, that the non-periodic term F_0 in the bottom friction F_b is given by

$$F_0 = fk\rho u_1 u_0, \tag{10}$$

where f is a factor which is a function of u_1/u_0.

$$\left.\begin{aligned} &\text{When } u_1/u_0 < 1, \quad f = \left(\frac{u_0}{u_1} + \frac{u_1}{2u_0}\right). \\ &\text{When } u_1/u_0 > 1, \quad f = \frac{1}{\pi}\left[\left(\frac{2u_0}{u_1} + \frac{u_1}{u_0}\right)\sin^{-1}\frac{u_0}{u_1} + 3\left(1 - \frac{u_0^2}{u_1^2}\right)^{\frac{1}{2}}\right]. \\ &\text{When } u_1/u_0 \gg 1, \quad f = 4/\pi = 1{\cdot}273. \end{aligned}\right\} \tag{11}$$

Considering the mean current from surface to bottom, the equation of motion is

$$\frac{\partial \bar{u}}{\partial t} + \frac{\bar{u}\,\partial \bar{u}}{\partial x} = -g\frac{\partial \zeta}{\partial x} + \frac{F_s - F_0}{\rho h}. \tag{12}$$

The equation of continuity takes the form

$$\frac{\partial}{\partial x}(A\bar{u}) + B\frac{\partial \zeta}{\partial t} = 0. \tag{13}$$

In the steady state, i.e. when $\partial \bar{u}/\partial t = \partial \zeta/\partial t = 0$, it may be shown that the difference in level between end-sections at $x = 0$ and $x = L$ may be expressed as

$$\Delta\zeta \equiv \zeta_L - \zeta_0 = \Delta\zeta_1 + \Delta\zeta_2 + \Delta\zeta_3, \tag{14}$$

where

$$\Delta\zeta_1 = \left[1 + \frac{1}{2(1+\beta)}\right]\frac{F_s}{\rho g}\int_0^L \frac{\mathrm{d}x}{h}, \tag{15}$$

where β is defined by

$$\beta = \frac{3N}{fkhu_1}. \tag{16}$$

As discussed in the previous paper (Bowden 1953) it is assumed that β remains constant, although N, f, k, h and u_1 may vary along the channel. A probable numerical value is $\beta = 3$.

$\Delta\zeta_1$ is a function of the wind stress only, for a given channel.

$$\Delta\zeta_2 = -\frac{\beta}{(1+\beta)}\frac{k}{g}\int_0^L \frac{fu_1\bar{u}}{h}\,\mathrm{d}x. \tag{17}$$

$\Delta\zeta_2$ is a function mainly of the mean flow $\bar{u}$, for a given channel, but may depend also on wind stress to the extent that f depends on the total velocity at the bottom.

$$\Delta\zeta_3 = -\frac{(\bar{u})_L^2}{2g}\left[1 - \left(\frac{A_L}{A_0}\right)^2\right]. \tag{18}$$

$\Delta\zeta_3$ is the loss in dynamic head due to the contraction of the channel, and depends on the mean flow.

In the steady state and when the term $u\,\partial u/\partial x$ is negligible, it follows directly from the equations of motion that

$$\rho g \frac{\partial \zeta}{\partial x} = \frac{\partial F}{\partial z} \quad \text{and} \quad \rho g h \frac{\partial \zeta}{\partial x} = F_s - F_0.$$

Hence

$$F = F_0 + (F_s - F_0)\frac{z}{h},$$

and since

$$F = \rho N\,\partial u/\partial z,$$

it follows that

$$\bar{u} = u_0 + \frac{h}{6\rho N}(F_s + 2F_0).$$

Using (10) to eliminate F_0, and the definition of β given by (16), the drift velocity at the bottom may be expressed as

$$u_0 = \frac{\beta}{(1+\beta)}\bar{u} - \frac{1}{2(1+\beta)}\frac{F_s}{fk\rho u_1}. \tag{19}$$

Suppose that the dimensions of a channel are known at each of a number of sections, at various values of x_1 and also the corresponding values of u_1. Then the integral in equation (15) may be computed directly and hence $\Delta\zeta_1$ for given values of β and F_s. To compute $\Delta\zeta_2$, a value of $\bar{u}$ at the initial section is assumed, and also a value of F_s, because of its effect on u_0 and so on f. An approximate value of f must be estimated and used in (19) to find u_0. Then f may be adjusted according to the value of u_1/u_0 and a better approximation found. This must be done for each section and $\Delta\zeta_2$ found by numerical integration of equation (17) from section to section.

In applying the method to the region under consideration, Sections 1 to 8 of the previous paper were taken on the English Channel side of the Straits of Dover. On the North Sea side five additional sections, 2*a* to 6*a*, were taken, the data for which are given in table 6.

Table 6. Sections across the southern North Sea

section no.	from	to	distance along channel x (km)	width B (km)	mean depth h (m)	area A (km^2)	amplitude of tidal current u_1 (cm/s)
1	St Margaret's Bay	Sangatte	0	35	35·3	1·20	110
2*a*	North Foreland	Gravelines	23	62	24·5	1·53	95
3*a*	Knock Deep	Furnes	46	93	25·1	2·32	95
4*a*	Harwich	Ostend	69	113	27·2	3·08	75
5*a*	Orford Haven	Zeebrugge	92	134	28·5	3·82	70
6*a*	Aldeburgh	Domburg	115	145	30·6	4·44	75

$\Delta\zeta_3$ is found to be negligible compared with $\Delta\zeta_1$ and $\Delta\zeta_2$. For $\bar{u} = 100$ cm/s across section 1, the narrowest part of the Straits, for example, $\Delta\zeta_3 = -4·8$ cm from the Shoreham/Dieppe section to section 1, but the level recovers with the opening out of the channel, so that from the Shoreham/Dieppe to the Aldeburgh/Domburg sections the difference in level is only $\Delta\zeta_3 = -1·5$ cm.

In computing $\Delta\zeta_1$ and $\Delta\zeta_2$, β has been taken at 3 throughout. Then with the other numerical values given above, equation (15) leads to

$$\Delta\zeta_1 = 9·4 F_s$$

when $\Delta\zeta_1$ is in cm and F_s in dyn/cm². If the stress is related to the wind velocity by a quadratic law,

$$F_s = \gamma^2 \rho_a W^2 \cos\phi.$$

Taking

$$\rho_a = 1{\cdot}25 \times 10^{-3}\,\mathrm{g/cm^3},$$

$$\Delta\zeta_1 = 117{\cdot}5\gamma^2 W^2 \cos\phi, \tag{20}$$

giving $\Delta\zeta_1$ in cm when W is in m/s.

$\Delta\zeta_2$ was computed for values of $\bar{u}$ across section 1 of 25, 50 and 100 cm/s and for $F_s = 0$, 2·5 and 5·65 dyn/cm² (values to be expected for winds of velocity 0, 10 and 15 m/s along the channel, if $\gamma^2 = 2\times 10^{-3}$). As shown by table 7, the influence of F_s on $\Delta\zeta_2$ is small. With sufficient accuracy, the results in table 7 may be represented by the linear relation

$$\Delta\zeta_2 = -380k\bar{u}, \tag{21}$$

where $\Delta\zeta_2$ is in cm and $\bar{u}$ in cm/s.

Table 7. Values of $\Delta\zeta_2/k$ in cm $\times 10^3$

$\bar{u}$ (cm/s)	F_s (dyne/cm²) 0	2·5	5·65
25	9·25	9·25	9·2
50	18·7	18·65	18·6
100	—	38·6	38·5

From (20) and (21), the total difference in level is given by

$$\Delta\zeta = \Delta\zeta_1 + \Delta\zeta_2 = 117{\cdot}5\gamma^2 W^2 \cos\phi - 380k\bar{u}. \tag{22}$$

This equation may be compared with the empirical equation (6) of § 6·4, derived from the observations on the St Margaret's Bay–Sangatte cable, i.e.

$$V = 0{\cdot}37 W^2 \cos\phi + 0{\cdot}70\Delta\zeta. \tag{6}$$

Identifying V with $\bar{u}$, and remembering that $\Delta\zeta$ as defined in § 6·4 is of opposite sign to that in the present notation, (14) may be put in the form

$$\Delta\zeta = 0{\cdot}53 W^2 \cos\phi - 1{\cdot}43\bar{u}. \tag{23}$$

Comparing (22) and (23) gives the following values for the coefficient of wind stress, γ^2, and the coefficient of bottom friction, k,

$$\gamma^2 = 4{\cdot}5 \times 10^{-3},$$

$$k = 3{\cdot}8 \times 10^{-3}.$$

It is interesting to note that these values for the two coefficients are approximately equal, although each is nearly twice the value to be expected. The general evidence available indicates a value of approximately $2{\cdot}5\times 10^{-3}$ for the coefficient of stress either between the air and a water surface or between flowing water and a fixed bed. In particular, reference may be made to values of $2{\cdot}5\times 10^{-3}$ and $1{\cdot}7\times 10^{-3}$ for γ^2 found by Rossiter (1954), by considering conditions at two stages of development of the storm surge of 31 January and 1 February 1953 in the North Sea. Allard (1952) found values of k of $2{\cdot}5\times 10^{-3}$ for the western part and $2{\cdot}05\times 10^{-3}$ for the eastern part of the English Channel, from the rate of dissipation of tidal energy, and Hansen (1952) gave $k = 3{\cdot}0 \times 10^{-3}$ for the North Sea. Grace (1937), by considering the relations between surface gradients and tidal currents, found $k = 2{\cdot}4 \times 10^{-3}$ at a section across the narrowest part of the Straits of Dover, but $k = 6{\cdot}3 \times 10^{-3}$ at a section 20 km to the south-west.

There are several possible explanations for the high apparent values of γ^2 and k, of which the following seem the most likely.

(1) The mean wind velocity over the sea surface in the region may have been systematically underestimated by taking the records for Lympne. There are few coastal stations providing anemometer records and no regular instrumental observations at sea. For limited periods, the Lympne records were compared with those from Dover and Manston and from certain stations on the French coast, but it seemed that the Lympne records alone would give as good a representation of the mean wind over the Straits as any other combination of the available records.

(2) The 25 h means of flow may not represent a sufficiently good approximation to steady state conditions for the theoretical treatment to be applicable. This point is discussed further in § 7·2 in connexion with the rate of response of the current to a change in wind stress or in surface gradient.

It appears unlikely that errors greater than about 10 % of their maximum values would occur in the rate of flow as determined by the cable measurements or in the differences of sea level derived from the tide gauge records.

7·2. *Variable state*

It is probable that the region of sea under consideration is seldom in a steady state and then only for a short time. An attempt to treat the changing state dynamically, however, presents several difficulties. In equation (12), a determination of $\partial \bar{u}/\partial t$ from the records of flow is possible, provided the tidal constituents have been eliminated adequately. The variation of F_s could be estimated from the observed changes in wind velocity, but F_0 is indeterminate, since it depends on the velocity near the bottom and it is possible to estimate the relation between bottom velocity and mean velocity only in the steady state.

$\partial\zeta/\partial x$ is an uncertain term, since, in a channel of such variable cross-section, the slope must vary considerably along the channel and tide gauge records are available only for widely separated positions on the coasts. In the steady state the value of $\partial\zeta/\partial x$ at the cable section is related to $\Delta\zeta$ between the end sections by a constant factor, but this is not true in changing conditions.

For these reasons, no attempt has been made to analyze the dynamics of the varying flow step by step. Several type problems have been treated and a rough quantitative comparison made with the results of the observations. In the previous paper (Bowden 1953), the rate of growth of a current in a uniform channel, due to the onset of a steady wind stress, was considered, the surface remaining horizontal. In the present notation, the solution may be written as follows.

When $t<0$, $F_s = 0$, $u = 0$.

When $t>0$, $F_s =$ constant.

Then
$$\bar{u} = \frac{F_s}{fk\rho u_1}\left\{1+\frac{\nu}{2}-2\sum_{n=1}^{\infty}\frac{\nu(\alpha_n^2+\nu^2)\sin\alpha_n}{\alpha_n^3\{\alpha_n^2+\nu(1+\nu)\}}\,e^{-\alpha_n^2 P}\right\},$$

where $\nu = fkhu_1/N = 3/\beta$, $P = Nt/h^2$, and α_n ($n = 1, 2, 3, \ldots$) are the positive roots of

$$\alpha \tan \alpha = \nu.$$

The corresponding solution for the growth of a current due to the setting up of a constant-surface gradient, the surface stress remaining zero, may be shown to be as follows:

When $t<0$, $G = 0$, $u = 0$, where $G = -g\partial\zeta/\partial x$.

When $t>0$, $G =$ constant.

Then
$$\bar{u} = \frac{Gh}{fku_1}\left\{1+\frac{\nu}{3}-2\sum_{n=1}^{\infty}\frac{\nu^2\tan\alpha_n}{\alpha_n^3\{\alpha_n^2+\nu(1+\nu)\}}\,\mathrm{e}^{-\alpha_n^2 P}\right\},$$

where ν, α_n and P have the same meanings as before.

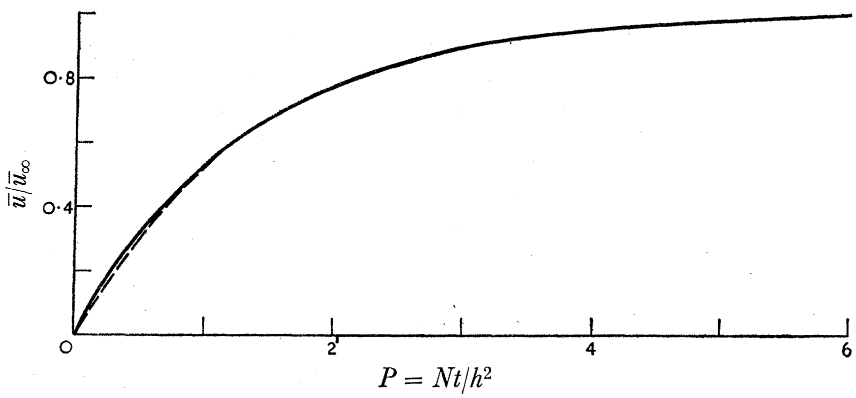

FIGURE 17. Response of mean current $\bar{u}$ to steady gradient (full line) or steady wind stress (broken line) set up at time $t = 0$.

The two solutions of $\bar{u}$, relative to its final value as $t\to\infty$, are plotted in figure 17. In the computation ν has been taken as 1, corresponding to the value $\beta = 3$, used in § 7·1. It is seen that the two curves are very similar.

The approach to the steady state depends on the parameter $P = Nt/h^2$. A similar parameter, namely, $1{\cdot}272Nt/h^2$, was found by Proudman & Doodson (1924) in the rate at which the surface of water in a closed rectangular basin would respond to a change in atmospheric pressure or surface wind stress. Proudman (1954) has suggested that this is a general result for the elevation ζ, whether the state is generated by wind or atmospheric pressure, and is independent of the distribution of current with depth. The problem treated by Proudman & Doodson was a more difficult one than that treated here, since both the elevation and current were unknown, but related, variables. Since a closed basin was assumed, however, the mean current in the steady state was zero. In the present problem, only the current appears as an unknown variable, since the surface gradient is taken as specified. This corresponds to the case of a narrow channel connecting two large seas, the flow through the channel being dependent on the difference in level in the two seas, but these levels being unaffected, to a first approximation, by the flow through the channel.

Another case which has been considered is the response of the current to a surge of gradient which rises to a maximum and then decays, according to the conditions:

when $t<0$ and $t>T$, $G = 0$;

when $0<t<T$, $G = \dfrac{G_0}{2}\left(1-\cos\dfrac{2\pi t}{T}\right)$.

The solution to this problem has been found by the method given by Carslaw & Jaeger (1947), using Duhamel's theorem, with the following result:

$$\bar{u} = \frac{G_0 h}{f k u_1} \sum_{n=1}^{\infty} \frac{\nu^2 \tan \alpha_n}{\alpha_n^3 \{\alpha_n^2 + \nu(1+\nu)\}} \times I_n,$$

where for $0 < t < T$,

$$I_n = 1 - \frac{e^{-\alpha_n^2 P}}{1 + \dfrac{\alpha_n^4 P_0^2}{4\pi^2}} - \frac{\cos \dfrac{2\pi P}{P_0} + \dfrac{2\pi}{\alpha_n^2 P_0} \sin \dfrac{2\pi P}{P_0}}{1 + \dfrac{4\pi^2}{\alpha_n^4 P_0^2}},$$

and for $t > T$,

$$I_n = \frac{e^{-\alpha_n^2 (P - P_0)} - e^{-\alpha_n^2 P}}{1 + \dfrac{\alpha_n^4 P_0^2}{4\pi^2}},$$

where $P = Nt/h^2$ as before and $P_0 = NT/h^2$.

The solution depends on the parameter P and also on another parameter P_0, characterizing the time scale of the variation in the gradient. Curves of $\bar{u}$ as a function of P for various values of P_0 are shown in figure 18. In all cases the maximum current lags behind the maximum gradient and the current decreases exponentially after the gradient has become zero. When $P_0 \gg 1$, the ratio of the peak current to the peak gradient approaches the value corresponding to steady-state conditions. For small values of P_0 the peak current is considerably less. When $P_0 = 2$, for example, the peak current is 0·46 the steady-state value.

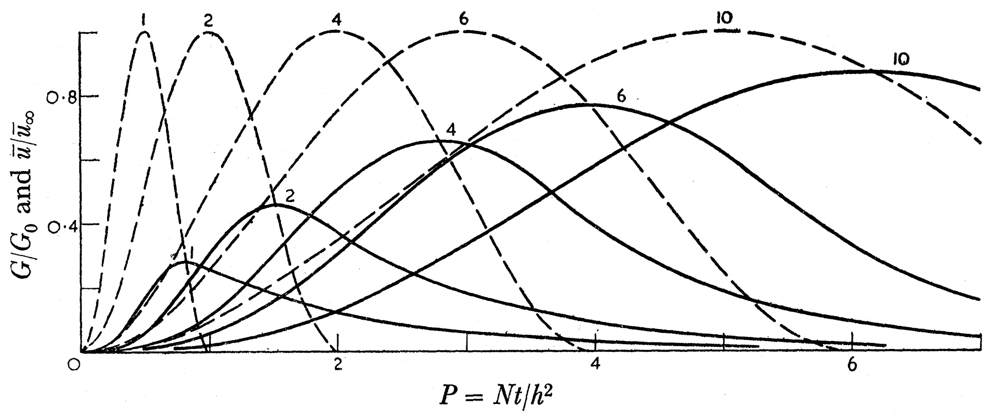

FIGURE 18. Response of mean current (full line) to a changing gradient (broken line). Values of P_0 are indicated against the curves.

For the Straits of Dover, $h = 35$ m and a reasonable value of N would be of the order of 10^3 cm²/s (see Proudman 1953, p. 314). Then $P_0 = 0{\cdot}30T$, when T is in hours. $P_0 = 4$, for example, when $T = 13{\cdot}3$ h, and with this value, the peak current would reach $\frac{2}{3}$ of its steady state value and would lag about $2\frac{1}{2}$ h behind the peak gradient. If $T = 26{\cdot}6$ h, $P_0 = 8$, the peak value would be 0·83 with a time lag of about $3\frac{1}{2}$ h. With increasing values of T, the time lag does not appear to increase very rapidly.

The corresponding problem for a variable wind stress has not been treated but, from the previous examples, it seems likely that the time relations would be very similar.

The above results may be compared with the features shown by the detailed analysis of the currents, wind and gradients during periods of strong flow, given in § 6·6. From figures

13, 14 and 16, the current peaks lag behind the corresponding peaks in the $W^2 \cos\phi$ or $\Delta\zeta$ curve by 0 to 6 h and there are indications of the slower decay of the current after the wind stress or gradient producing the flow has decreased. It is of some interest to compare the peak values of the current with those computed from equation (14), derived from the 25 h means. The results are shown in table 8 for six occasions on which the peak flow exceeded 50 cm/s. On 21 September and 7 November 1953, the observed current was less than the computed value, but on the other four occasions the observed current was greater. The discrepancies between the observed and computed values are considerable but this is not, perhaps, surprising since the computed values are based on the assumption of steady state conditions.

TABLE 8. PEAK VALUES OF CURRENT SURGES

date	hour	V_0 observed (cm/s)	$W^2 \cos\phi$ (m/s)^2	$\Delta\zeta$ (cm)	V_c computed (cm/s)	$\lvert V_0\rvert - \lvert V_c\rvert$ (cm/s)
1953: 21 Sept.	18.00	83	216	48*	113	−30
27 Oct.	12.00	84	52	38	46	38
1 Nov.	21.00	99	83	68	78	21
4 Nov.	06.00	51	55	26	38	13
7 Nov.	15.00	54	108	37	66	−12
1954: 3 Jan.	21.00	−97	−24	−102†	−82	15

* $\Delta\zeta$ for Newhaven/Dieppe–Lowestoft/Flushing.
† $\Delta\zeta$ for Shoreham–Lowestoft/Flushing.

The surges of wind or gradient in figures 13, 14 and 16 had a duration of the order of 24 h, with the exception of that reaching a peak on 1 November, which lasted for nearly 48 h. With the numerical values taken above, the theory indicates that the peak value of current for a surge of 24 h duration would be about 80 % of the steady state value. Since many of the lesser variations in current are probably of a similar period, it seems possible that the 25 h mean values of current may be less than those corresponding to a steady state by a similar amount. This would account, in part, for the high values deduced for the stress coefficients γ^2 and k.

The general features of the current surges and of the 25 h mean values would appear to be consistent with those anticipated from dynamical theory, and the values deduced for the coefficients of wind stress and bottom friction, although rather high, are of the order expected. To obtain a better quantitative understanding of the flow through the Straits of Dover and of the processes on which it depends, analyses of many more examples under various conditions would be desirable.

This investigation has been made possible by the co-operation of many people and organizations. I wish to acknowledge my indebtedness to the following: The Post Office Engineering Department, for allowing the use of the telephone cables and for the willing co-operation shown by the Headquarters staff and those at the repeater stations; Commander W. I. Farquharson and the staff of the Tidal Branch of the Hydrographic Department; the Meteorological Office; the Port of London Authority, the Dover Harbour Board, the Shoreham Harbour Trustees, the Marine Department of British Railways, the Netherlands Rijkswaterstaat, and the French Service Central Hydrographique for the loan of tide gauge records. I also wish to express my gratitude to Dr G. E. R. Deacon, F.R.S., and my colleagues in the National Institute of Oceanography, particularly Miss M. Irving, who has done the greater part of the measuring and computational work in the analysis of the records.

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